Rhind Mathematical Papyrus

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INTRODUCTION[1]

The Rhind Mathematical Papyrus was found at Thebes in the ruins of a small building near the Ramesseum. It was purchased in 1858 by A. Henry Rhind and after his death came into the possession of the British Museum. It is a copy that was made by a scribe named A'h-mosè and begins with a rather ambitious statement of what the author proposes to do. The date at which this copy was made is indicated in the old Egyptian method by the year of a certain pharaoh named 'A-user-Rê',[2] who has been identified as one of the Hyksos dynasty, living approximately 1650 B.C. The scribe further says that it is in likeness to older writings, of the time of king Ne-ma'et-Rê' (Amen-em-hêt III) who reigned from 1849 to 1801 B.C.[3] The papyrus is written in hieratic[4] and originally it was a single roll nearly 18 feet long and about 13 inches high, but it came to the British Museum broken apart, and with a number of fragments missing, the most important of which have been found in the possession of the New York Historical Society. A lithographic facsimile of the papyrus was published by the British Museum in 1898, but the photographs which I have reproduced in the second volume of this work represent it as it appears to-day. The fragments found in New York were brought over

[1] In referring to a publication listed in the Bibliography, I shall generally give with the name the year of publication, and if there is more than one publication listed under the same name in a given year, a second number will indicate the one referred to. But all references to Peet will be to his edition of the Rhind Papyrus and all references to Gunn to his review of Peet, both listed under Peet, 1923, 2, and only the number of the page or plate will be given. Nearly all references to Griffith will be to the series of articles listed under 1891 and 1892, and in these references only the numbers of the volume and page will be given.

[2] For the pronunciation of Egyptian words see the note at the beginning of the Literal Translation. Only consonants were written during most of Egyptian history, but the vowels supplied by us are sometimes inferred from those of a late period. Each vowel indicates a syllable. Hyphens are used in compound words. The sign ' represents a consonant which we need not pronounce. Thus the god's name "Rê'" at the end of a royal name may be pronounced like the English word "ray."

[3] For chronology see J. H. Breasted, *A History of Egypt*, second edition, New York, 1911, page 597.

[4] Egyptian writings that have come down to us are in four forms: (1) hieroglyphic, used in formal writings such as temple and tomb inscriptions; (2) hieratic, a more cursive form, both of these forms appearing at the beginning of Egyptian history; (3) demotic, which appeared about 800 B.C., and replaced the hieratic for everyday use; and (4) Coptic, in which the Egyptian language was written with the Greek alphabet. This last form arose in the third century A.D. and survives to-day in the liturgy of the Coptic church.

by Edwin Smith with his papyrus (see J. H. Breasted, *The Edwin Smith Papyrus*, Oxford, in press). This was discovered in 1922 when Professor Newberry, the English Egyptologist, suspected that they were some of the missing pieces and brought tracings of them to Professor Peet (see Bibliography under Ahmes, 1898). Peet publishes a plate (Plate E) on which with much skill he has arranged nearly all of the writing on them, and I have copied this writing on my plates, placing it with the problems to which it belongs, and including its translation with the translation of those problems. I have also put along with the photographs of the papyrus a photograph of the fragments placed in their proper positions, except for a few small pieces whose positions have not yet been determined.

The rest after the introductory paragraph may be divided into three parts,—an arithmetical part, a geometrical part, and a part containing a collection of miscellaneous arithmetical problems. In my judgment the arithmetical part is more exact than the geometrical part, and exhibits more reasoning power. The geometrical part reveals considerable knowledge of geometrical facts useful for the determination of volumes, areas, and line relations. The section of miscellaneous problems is interesting because it gives us some knowledge of Egyptian customs, their method of trading, of raising taxes, of feeding animals, and of fixing the comparative values of different foods and drinks by the amount that can be made from a unit of material. It contains also various methods of solution, some of which have come down to the present day and are found in our arithmetics.

EGYPTIAN ARITHMETIC

Numbers and Operations

We have very few traces of Egyptian arithmetic from a date earlier than the original sources of the Rhind papyrus. But before this date there was a long period of Egyptian civilization, and we may well believe that for more than a thousand years there had been a slow development of the elaborate system before us.

The Egyptians apparently conceived of two kinds of numbers, an ascending series from 1 to 1,000,000 of numbers that we call integers, and a corresponding descending series consisting of $\frac{2}{3}$ and reciprocal numbers or unit fractions.[1]

For integers they had:

1. A well-defined decimal notation, without, however, the positional device that renders our modern notation so simple;

2. A thorough understanding of the four arithmetical operations, —addition, subtraction, multiplication, and division.

Addition and subtraction were easily accomplished. Direct multiplication by integers was generally confined to the multipliers 2 and 10; that is, in cases where the process was explicit, and it may be presumed that in all cases[2] they multiplied by repeated doubling, or occasionally multiplying by 10, and adding the products formed from multipliers that would make up the given multiplier. This is not very different from our modern way, where we multiply by the unit figure of our multiplier, and then by the ten figure, and so on, and then add the re-

[1] These are the same as our fractions with numerator 1, and it will be convenient, for the most part, to speak of them as fractions. They are sometimes called fundamental fractions (Stammbrüchen). When the Egyptians wish to express a number that is not a single unit fraction, they use a combination of such fractions. Thus for our $1\frac{3}{4}$, they say $1\ \frac{1}{2}\ \frac{1}{4}$; for 2 times $\frac{1}{5}$, $\frac{1}{3}\ \frac{1}{15}$, and so on. Such an expression we may speak of as a fractional number or fractional expression; it was understood that the fractions, or the whole number and fractions, were to be added, and I shall always write them as do the Egyptians without any sign of addition, just as we write $1\frac{1}{2}$. A given number could be expressed as a sum of unit fractions, or as the sum of a whole number and such fractions, in an infinite number of ways, and sometimes the expression for a given number is varied, but the fractions of an expression must all be different fractions, and very rarely were they allowed to have a sum equal to or greater than 1.

[2] Many times the details of the multiplication are omitted in the papyrus and we cannot say with certainty what was done in every case, but it is fair to make inferences if we take into account all of the cases in which the details are given.

sults. In most of the multiplications all through the papyrus the author checks those multipliers that he is to use.[1] Division was performed by successive multiplication of the divisor until the dividend was obtained.[2]

For the numbers of the descending series they had:

1. A notation which for reciprocal numbers was nearly like their notation for integers, these numbers being distinguished from integers by having a dot in hieratic and the sign ⬭ in hieroglyphic written over them, except that the first three had special signs in hieratic, and the first, $\frac{1}{2}$, in hieroglyphic also;

2. Special devices for addition and subtraction, because it was necessary to express the result using only integers and different unit fractions (see page 3, footnote 1); and for multiplication, because, apparently the only fractions that they could use as direct multipliers were $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{10}$.

Addition and subtraction will be explained below. In multiplication they generally took $\frac{2}{3}$ and then halved to get $\frac{1}{3}$,[3] and they took $\frac{1}{10}$

[1] For the multiplication of 19 by 6 the Egyptians would say,

	1	19
	\2	38
	\4	76
Total,	6	114,

the two multipliers which make up 6 being checked; and for 12 times 23,

	1	23
	\10	230
	\2	46
Total	12	276.

See page 52.

In Problem 79, 2801 is multiplied by 7 in the same way,

1	2801
2	5602
4	11204
Total	19607.

[2] Dr. Reisner tells me that when the modern Egyptian peasant wishes to divide things of some sort among a number of persons he first distributes a certain number, say 5, to each, and then perhaps 2, and so on.

[3] Peet, page 20. Examples are in Problems 25, 29, 32, 38, 42, 43, and 67; besides 8 and 16–20, and many times in the table at the beginning. $\frac{1}{3}$ is given without any $\frac{2}{3}$ in Problems 28, 29, 32, 39, 42 (line 8 of the multiplication), and several times in 35–37. We may suppose that in these cases the author did not intend to put in the work, but only the result. Gunn says (page 125) that "The Egyptian, like everyone else, had ultimately no way of arriving at $\frac{2}{3}$ but via $\frac{1}{3}$." This is not quite true, for, whatever $\frac{2}{3}$ meant to him, he knew for a fact that it is the reciprocal of $1\frac{1}{2}$ (see, for example, page 75, footnote, also Gunn, page 129), and it was possible for him to make a table for $1\frac{1}{2}$ and read it backwards, just as he sometimes reads backwards the table for the division of 2 by odd numbers (page 20, footnote 2). A number not found in his table (for example, a number that is not a multiple of 3) could be obtained by putting other numbers together, using $\frac{2}{3}$ for $\frac{2}{3}$ of 1, and $1\frac{1}{3}$ for $\frac{2}{3}$ of 2. This is explained by Gunn himself on page 125.

and then doubled to get 1/5.[1] The reciprocals of other numbers were sometimes used as multipliers, when the numbers themselves had appeared in previous multiplications that could be transformed into the multiplications desired.[2] In particular, they often used the fact that the reciprocal of a number multiplying the number itself gives 1.

Egyptian division might be described as a second kind of multiplication, where the multiplicand and product were given to find the multiplier.[3] In the first kind of multiplication, the multiplier, being given, can be made up as a combination of the multipliers that were generally used, and the corresponding combination of products would be the required product. When it was the product that was given along with the multiplicand, various multipliers would be tried, 2, 10, and combinations of these numbers, or combinations of the fractions 2/3, 1/2, and 1/10, and from the products thus obtained the Egyptians would endeavor to make up the entire given product.[4] When they succeeded in doing this the corresponding combination of multipliers would be the required multiplier. But they were not always able to get the given product at once in this way, and in such cases the complete solution of the problem involved three steps: (a) multiplications from which selected products would make a sum less than the required product but nearly equal to it; (b) determination of the remainder that must be added to this sum to make the complete product; and (c) determination of the multiplier or multipliers necessary to produce this remainder. The multipliers used in the first and third steps made up the required multiplier. The second step was called completion and will be explained below. For the third step they had

[1] Peet, page 19, footnote 1. Examples occur in Problems 21, 22, and 35.

[2] Thus in Problem 34, 4 times 1 1/2 1/4 is equal to 7, and therefore 1/7 of 1 1/2 1/4 is equal to 1/4. There are some striking illustrations in Problem 38. See also Problems 33 and 70.

[3] The usual form of expression was "Operate on (Make, Make the operation on) . . . for the finding of" . . . (Problems 21, 22, 26, 30, 56, etc., see Literal Translation). In the table at the beginning of the papyrus the Egyptian says several times, "Call 2 out of" some number, and this form of expression is used also in Problems 35, 37, 38, 63, 66, and 67. But the point of view is the same, the idea being to get 2 as a result of operating on the number (see Gunn, page 124), and the process is the same as when the former expression is used. In Problems 1–6 and in 65 the author speaks of "making" a certain number of loaves "for" a certain number of men, and in 54 and 55 he uses the expression "Take away a certain area from a certain number of fields," that is, by taking an equal amount from each field (see page 96), but these are forms of expression used for the statement of the problems and not technical expressions for any kind of division. The expression "Call 2 out of" . . . should be particularly noticed, for we should naturally take the numbers the other way. We say that "3 will go into 6 twice" and we can imagine some one saying "Call 3 out of 6," but the Egyptian says "Call 6 out of 3."

[4] This happens in Problems 34–38, also in 69. See notes to 34 and 37.

a definite process which they generally used. The remainder, being a small number, would consist of one or more reciprocal numbers. For one of these numbers the third step may be expressed by the rule: To get the multiplier that will produce the reciprocal of a given whole number as a product multiply the multiplicand by the number itself and take the reciprocal of the result of this multiplication.[1] If, for example, we wish to multiply 17 so as to get $\frac{1}{3}$, we take 3 times 17, which is 51, and then we can say that $\frac{1}{51}$ of 17 equals $\frac{1}{3}$.[2] See page 17. In both kinds of multiplication the product is formed by adding certain multiples of the multiplicand, and so the multiplicand and product, instead of being mere numbers, may be things of some kind, while the multiplier must always be a mere number.

On the other hand, the Egyptians could not solve directly the problem of finding the multiplicand when the multiplier and product were given. At the present time we scarcely think of this as a new problem; in fact, we never notice the distinction between multiplier and multiplicand, often saying that we multiply two numbers together. Even when some of our numbers represent things, we always think of them as mere numbers while we are multiplying or dividing. The Egyptian method emphasized this distinction, even when all the quantities were mere numbers. However, it was known that the product then would be the same if the multiplier and the multiplicand were interchanged, and sometimes, even in the first kind of multiplication, they were interchanged, simply because this would make the multiplication easier.[3] When the multiplier and product were given the Egyptians usually made this interchange, thinking of all their quantities as mere numbers;[4] that is, they changed their problem into one in which the given multiplier became the multiplicand and the required multiplicand was to be obtained as a multiplier, only at the end to be interpreted as the problem required. Problems 30–34 are particularly good illustrations of this method; also some of the *pefsu* problems (69–78), where the notation sometimes

[1] To use Gunn's expression (pages 129 and 134) we might say, "Multiply by the number itself and dot the result."

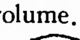
[2] This rule is illustrated in Problems 34 and 70, and all through the table for the division of 2 by odd numbers. The formation of this table is for each number a multiplication problem of the second kind, and, except for the cases of 3, 35, and 91, the three steps of the process are always present.

[3] This is done in the third step in the solutions of Problems 24–27.

[4] In Problem 55, instead of regarding the number of the given product as a mere number, the author regards the given multiplier, now taken as a multiplicand, as representing things of the same kind as the product. See notes to that problem.

tells us which numbers are to be regarded as mere numbers, and which represent things of some kind.[1]

But sometimes they wished to keep in mind the nature of the quantities in the problem, and then, instead of solving it as a problem in multiplication, they used a process of trial that has been called "false position" and will be explained below.

The Egyptians commenced their descending series with the quantity $\frac{2}{3}$, and it is interesting to note that in the papyrus of Akhmîm (Baillet, 1892), written in Greek about 600 A.D., the Egyptian system of unit fractions is still used, and with the same apparent exception of $\frac{2}{3}$. Griffith (volume. 16, page 168), regarding the short line in the hieroglyphic sign  as a half of the longer line, saw in it a symbolic expression for 1 divided by $1\frac{1}{2}$, but it is known now that at first the two strokes were of the same length, so that this idea applies, if at all, only to later Egyptian times.[2]

Special Processes Emphasized in the Rhind Papyrus

I will mention three special processes:

1. A method in which a fractional expression is applied to some particular number;

2. The solution of problems by false position;

3. A process of completion used for determining the amount to be added to an approximation to a given number in order to get the number.

It may be well at this point to explain these processes somewhat carefully.

1. If we wish to add two groups of fractions, say $\frac{1}{3}$ $\frac{1}{5}$ and $\frac{1}{5}$ $\frac{1}{7}$, we reduce them to the common denominator 105. Then as many times as each denominator has to be multiplied to produce the common denominator, so many times must the numerator of that fraction be multiplied to produce the new numerator, and these new numerators,

[1] In Problem 64 also this is shown by the notation. See page 30.

[2] Sethe (1916, pages 91 ff.) finds some traces among the Egyptians, as well as with other ancient peoples, of a system of complementary fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, . . . They called these expressions "the 2 parts," "the 3 parts," etc., the 2 parts being what is left after taking away $\frac{1}{3}$, and so with the others. Thus in the Hebrew Scriptures we read (Genesis 47, 24) that Joseph said to the Egyptians when they sold themselves and their land for food, "Ye shall give the fifth part unto Pharaoh and four parts shall be your own." $\frac{2}{3}$ is the only fraction of this system to be found in the Rhind papyrus. In the papyrus of Akhmîm the sign for $\frac{2}{3}$ is always preceded by the singular article, τὸ, as if it were one part (Baillet, page 19).

added together, will give the numerator of the required result, in this case 82, so that the sum of the fractions will be $\frac{82}{105}$.

Suppose that an Egyptian had the same problem. Since a fractional expression with the Egyptians was a sum of fractions, or of a whole number and fractions, we might think that he could add the two groups by writing down the four fractions together, as $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{7}$, but one law of the Egyptian fraction-system required that the fractions in an expression should all be different. To add the fractions, he would think of a number or of a group of things of some kind, to which he could suppose them to refer, a number or group such that each fraction of it will be a whole number or at least one that he can add to other numbers of the same kind. He might take 105, say 105 loaves, and suppose that he is to receive $\frac{1}{3}$ $\frac{1}{5}$ of them and also $\frac{1}{5}$ $\frac{1}{7}$ of them. He would say that $\frac{1}{3}$ of 105 loaves is 35 loaves, and $\frac{1}{5}$ is 21, making 56 loaves in the first group, and that $\frac{1}{5}$ is 21 loaves and $\frac{1}{7}$, 15, making 36 loaves in the second group, and that he will receive in all 82 loaves as his part of 105. But he could not say that he will have $\frac{82}{105}$ of them all. All that he can do is to seek to express the 82 loaves as an aggregate of parts—different parts—of 105 loaves.

To find what parts of 105 loaves will make 82, he would take fractional multipliers and seek to multiply 105 so as to get 82. This would be a multiplication of the second kind (see page 5). He might say[1]

1	105
\ $\frac{2}{3}$	70
$\frac{1}{3}$	35
$\frac{1}{30}$	3 $\frac{1}{2}$
\ $\frac{1}{15}$	7
$\frac{1}{10}$	10 $\frac{1}{2}$
$\frac{1}{5}$	21
\ $\frac{1}{21}$	5

Total $\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{21}$

But he might be able without going through a formal multiplication to separate 82 loaves into groups which he recognizes as parts of 105. Thus he might take 70, which is $\frac{2}{3}$ of 105, leaving 12, and these he could separate into 7 and 5, which are $\frac{1}{15}$ and $\frac{1}{21}$ of 105.

If now he finds in some way that 82 of the loaves will be $\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{21}$ of the 105 loaves, then he will know that $\frac{1}{3}$ $\frac{1}{5}$ plus $\frac{1}{5}$ $\frac{1}{7}$ make $\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{21}$ when applied to 105 loaves, and he will conclude that they always make $\frac{2}{3}$ $\frac{1}{15}$ $\frac{1}{21}$.

[1] The Rhind Papyrus contains multiplications for this purpose in Problems 21 and 22.

This example illustrates what I mean by "applying a fractional expression to some particular number."

Sometimes the Egyptian wishes to use this method with an expression involving a whole number as well as fractions. Thus in the first multiplication of Problem 32 of the Rhind Papyrus the author desires to select partial products that will add up to 2. Regarding all of them as referring to a group of 144 things of some kind, assuming perhaps that he has several such groups, he finds the values of these partial products as applied to 144 and seeks to make up in this way the number of things in two groups, that is, 288. The multiplicand, 1 $\frac{1}{3}$ $\frac{1}{4}$, makes one whole group and $\frac{1}{3}$ and $\frac{1}{4}$ of another, or 228 things in all. The next expression, 1 $\frac{1}{18}$, makes 152 things, and so for the others.

The examples in the papyrus seem to indicate that there was no definite rule for determining what number would be most convenient to take. Naturally it would be desirable to take a number for which it was easy to find the parts indicated by the given fractions. The number taken is often the largest number whose reciprocal is among them. Thus in many cases the parts are not all whole numbers, but are whole numbers and simple fractions. In Problem 33 one of the parts is [1] 13 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{14}$ $\frac{1}{28}$.

This method of applying an expression to a particular number was continued for many generations and is found in the papyrus of Akhmîm already mentioned. Rodet (1882, page 37) calls the number taken a *bloc extractif* out of which these fractions are drawn. His idea may be somewhat like that which I have expressed, but his explanation seems a little abstract, and the use of a technical term, while very convenient, makes the process seem more improbable for so ancient a people. Hultsch (1895) has a theory still more formidable. He supposes that the Egyptian introduces an auxiliary unit. In the example above he would say that the Egyptian, for purposes of addition, makes $\frac{1}{105}$ a new unit, in terms of which the given fractions become whole numbers that can be readily added. Peet (page 18) considers that the question is merely one of notation, that the Egyptian really did have the conception of a fraction with numerator greater than 1 but no notation for

[1] The problems in which there is evidence of the use of this method are 7–20, 21–23, 31–34, 36–38, and 76, and the case of 2 divided by 35 in the first table. In Problem 36 this method is used for a division process, and in the case of 2 divided by 35 for a multiplication, but generally it is used for the addition or subtraction of quantities. In all cases in the papyrus the only fractions that appear as parts of the number taken are $\frac{2}{3}$, $\frac{1}{3}$, and fractions whose denominators are powers of 2, except in Problem 33.

such fractions.[1] In fact, he claims that the theories of Rodet and Hultsch are essentially the same as the modern theory, "bloc extractif" and "auxiliary unit" being only other names for common denominator. Perhaps he would say that my "particular number" is only another name for common denominator, but it does not seem so to me. The idea of taking a number, solving the problem for this number, and assuming that the result so obtained holds true for any number, is exactly what the boy in school is inclined to do for all sorts of problems, and what the author of our handbook does in much of his work. In fact, he always takes a particular number, and when he has solved a problem he does not hesitate to take a more complicated one of the same kind and use the same method. I do not think that the idea of a common denominator or of a fraction with numerator greater than 1 is involved in the theory as I have explained it, even though the number used is the same number as our common denominator, and some of the numerical work is the same as when a common denominator is used.[2] When I say that $1/3$ of 105 is 35 and $1/5$ is 21, together making 56 things, there is no suggestion that $1/3$ is equal to $35/105$ and $1/5$ to $21/105$.

2. The process of false position (*positio falsa*) consists in assuming a numerical answer and then by performing the operations of the problem getting a number which can be compared with a given number, the true answer having the same relation to the assumed answer that the given number has to the number thus obtained.[3] In this method we see one point of distinction between arithmetic and algebra. In algebra a letter x represents exactly the answer, and its value is obtained by solving an equation. In this method we take a number

[1] Sethe, in his review of Peet (See Bibliography under Peet, 1923, 2) expresses emphatic dissent from this view. He says "Wherever there was the notion there would have been already established the word and the sign. Indeed, it would have been entirely incomprehensible that the Egyptians should have written $1/2$ $1/4$ or $1/3$ $1/12$ if they did not also read these words, and that the Greeks, Romans, and Arabs should have held on so long to the really circuitous reckoning with fundamental fractions if the so much simpler method of mixed fractions [fractions with numerator greater than 1] had already so early been known." See also Sethe, 1916, pages 60 and 62.

[2] It is just as the modern schoolboy in solving simultaneous equations by different methods will have the same numbers to multiply or divide although the theories of the different methods are different.

[3] We might say that this is proportion, but the words "false position," with their suggestion of the days when we had only arithmetic in our schools, give us a better realization of the Egyptian point of view. The rule was called by the Arabs, *ḥisab al-Khataayn*, and hence it appears in medieval Europe as *Elchataym*. It was also known by such names as *Regula falsi, Regula positionum, False positie*. See D. E. Smith, 1923, volume 2, pages 437–438.

which (usually) is not the answer, and get the answer by finding its relation to this number.[1]

False position[2] is used in a number of multiplication problems in which the multiplier and product are given and it is required to find the multiplicand, because by this method it is possible to keep in mind the nature of the quantities involved. Problems 24–27 exhibit the method very clearly although the required multiplicand in these problems is nothing more definite than "a quantity," and the given product, which must be a quantity of the same kind, is given simply as a number. It is instructive to compare this group of problems with Problems 30–34, which interchange multiplicand and multiplier. The problems of the two groups are problems of the same kind and all of them involve only "a quantity."

I think, however, that the *hekat* problems, 35, 37, and 38, show more clearly that this process enables us to keep in mind the nature of the quantities involved, and I will give in detail the reasoning of one of these problems as I understand it.

In Problem 35 a vessel filled $3\frac{1}{3}$ times with grain will make 1 *hekat* (about half a peck). The vessel contains so much grain and the *hekat* is so much grain. $3\frac{1}{3}$ times the former makes the latter. Here we have given the multiplier, $3\frac{1}{3}$, and the product, 1 *hekat* of grain, to find the multiplicand, which will be a certain portion of a *hekat* of grain. Assume a vessel that itself contains 1 *hekat;* this filled $3\frac{1}{3}$ times will give $3\frac{1}{3}$ *hekat*. Then the one that we have must bear the same relation to the one assumed, that the amount of grain that fills the former $3\frac{1}{3}$ times bears to the amount of grain that fills the latter $3\frac{1}{3}$ times; that is, that one *hekat* bears to $3\frac{1}{3}$ *hekat*. Thus we have to find the relation of one *hekat* to $3\frac{1}{3}$ *hekat*. Making $3\frac{1}{3}$ *hekat* the multiplicand and one *hekat* the product we get for the multiplier the number $\frac{1}{5}$ $\frac{1}{10}$. The amount of grain that the given vessel holds is then $\frac{1}{5}$ $\frac{1}{10}$ times the amount of grain that the one assumed holds; that is, it is $\frac{1}{5}$ $\frac{1}{10}$ times 1 *hekat*.[3]

[1] We might notice a point of similarity in this method and the method of applying fractions to a particular number, that in both methods we first assume a particular number or quantity. In Problem 76 the same assumed quantity, namely, 30 loaves, is used for both purposes. First the writer takes $\frac{1}{20}$ and $\frac{1}{30}$ as parts of 30 to show that the sum of these fractions is $\frac{1}{12}$, and then he takes 30 loaves as the basis of a false position process for determining the answer to the problem.

[2] False position is used in Problems 24–27, 40 and 76. It is used also in Problems 28, 29, and 35–38 as I have explained them.

[3] It is a little confusing to have 1 *hekat* come into the solution in two ways.

The process of false position can be applied to other problems besides those of division. This is well illustrated in Problem 40, and I will give an explanation of it at this point.

This problem is to divide 100 loaves among five men in such a way that the shares received shall be in arithmetical progression and that $\frac{1}{7}$ of the sum of the largest three shares shall be equal to the sum of the smallest two. The papyrus does not say that the shares shall be in arithmetical progression, but the solution shows that this is the intention. After stating the problem it simply says that the difference is $5\frac{1}{2}$ without explaining the method of obtaining this number, nor does it say that the smallest share shall be 1 nor even mention the smallest share. We may suppose that it is so natural to think of 1 as the smallest number that it does not occur to the author to mention it, even when he finally gets an answer in which the smallest number is not 1. Now to get the common difference he may have assumed first a common difference 1. The terms of the progression would be 1, 2, 3, 4, 5, the sum of the smallest two would be 3, and $\frac{1}{7}$ of the sum of the largest three would be 1 $\frac{1}{2}$ $\frac{1}{7}$ $\frac{1}{14}$, a difference between the two sides of 1 $\frac{1}{4}$ $\frac{1}{28}$. If he then assumed a common difference of 2, and hence the progression 1, 3, 5, 7, 9, he would find the sum of the smallest two terms to be 4, and $\frac{1}{7}$ of the sum of the largest three, 3, making a difference between the two sides of 1. In other words, for each increase of 1 in the assumed common difference he would find the inequality between the two sides reduced by $\frac{1}{4}$ $\frac{1}{28}$. To make the two sides equal he must multiply his increase 1 by as many times as $\frac{1}{4}$ $\frac{1}{28}$ is contained in 1 $\frac{1}{4}$ $\frac{1}{28}$, which is $4\frac{1}{2}$, and this added to the first assumed difference 1 makes $5\frac{1}{2}$ as the true common difference. This process of reasoning is exactly in accordance with Egyptian methods.[1]

To determine if the progression which he has obtained fulfills the second requirement of the problem, namely, that the number of loaves shall be 100, he proceeds as follows: Having the progression 1, $6\frac{1}{2}$, 12, $17\frac{1}{2}$, 23, he finds that the sum is 60 instead of 100. Therefore it is necessary to multiply by the factor that will produce 100 from 60,

[1] Peet (page 78) seems to think that this problem is like some of the inverse problems of the papyrus (see below, page 35); that the author, having a series with a common difference of $5\frac{1}{2}$, noticed in this series the relation between the sum of the smallest two terms and the sum of the largest three, and made up an inverse problem with 100 for sum instead of 60. But it is as difficult to see how he should discover this relation in a series that he has before him, as it is to see how he should find the common difference, $5\frac{1}{2}$, when he has given this relation.

namely, by $1\frac{2}{3}$. The true smallest term will then be $1\frac{2}{3}$ and we shall have the true division of loaves

$$1\ \frac{2}{3}\quad 10\ \frac{2}{3}\ \frac{1}{6}\quad 20\quad 29\ \frac{1}{6}\quad 38\ \frac{1}{3}.$$

This problem is notable because, while the Egyptian mathematician did not have such a thing as simultaneous equations, yet by methods which were within his knowledge he could sometimes obtain the result when there were two unknown quantities, as illustrated here.

The process of false position was employed by Diophantus and by Arabic writers, and has continued in use even down to our own day, being found in older arithmetics;[1] it was probably dropped from use about the time that algebra began to be generally taught in our schools.

3. The third of the special processes was a process of completion, used for determining the amount to be added when we have very nearly a given number. It was used especially in the second kind of multiplication as explained on page 5. Problems 21–23 are given as problems in completion and show the method of solving such problems. Thus in Problem 21 we have to complete $\frac{2}{3}$ $\frac{1}{15}$ to 1. To determine the answer, these fractions are applied to 15. $\frac{2}{3}$ $\frac{1}{15}$ of 15 make 11 and require 4 more to make the whole of 15. 4 is the same as $\frac{1}{5}$ $\frac{1}{15}$ of 15, and therefore $\frac{1}{5}$ $\frac{1}{15}$ is what is required to complete the given fractions to 1. The three problems are all solved in this way.[2]

I may add that there is another group of problems, 7–20, before which the author puts the words, "Example of making complete," but probably by mistake as these problems are all simple multiplications. See page 23, footnote 2.

Table of the Division of 2 by Odd Numbers

Inasmuch as the Egyptian mathematician performed his multiplications mostly by doubling or halving it was necessary that he should be able to double any numerical quantity, a reciprocal as well as a whole number. This could easily be done with the reciprocal of an even number, but for odd numbers it was convenient to have a special table. To determine the double of a reciprocal number was the same as

[1] See, for example, Benjamin Greenleaf, *The National Arithmetic*, revised, Boston, 1853, page 286. He calls it, "Single Position."

[2] In the formation of the table for the division of 2 by odd numbers completion is one of the processes for all but three of the numbers. In simple cases the completion may not have required elaborate calculations, but whenever the author did not know at once what were the fractions required, the procedure illustrated in Problems 21–23 would be used. In six cases the word "remainder" is put in, indicating the result of a completion process. These are for 2 divided by 17, 19, 23, 37, 41, and 53. Completion is also used in Problems 30–34.

to find by what the number itself must be multiplied to get 2, and so the problem becomes a multiplication problem of the second kind, a problem in which the given odd number is the multiplicand and 2 is the product. In the first eight pages of the papyrus these relations are obtained for all the odd numbers from 3 to 101. Theoretically the result can be put in an infinite number of forms; the forms given are generally the simplest. In the first part of the table the relations are worked out in detail in a sufficient number of examples to show the method.

In the papyrus the scribe places his answer to the left of the given number and the "reckoning" below. Hultsch (1895, page 4) and Peet (page 34) both regard the latter as the proof and not the solution. It is true that the interpretation of this as the solution leaves the table without proofs, but the proof would be somewhat like the solution and perhaps the author thought that one process would answer both purposes. The process is introduced by the word *seshemet*, put in with each number that comes at the top of a page. This word is used elsewhere in the papyrus only to introduce a solution, or the numerical work of a solution after a summary has been given in words.[1] Even if we are to regard the answer as coming before this numerical work, which is placed on lower lines, the arrangement would be like the arrangement in many other places where the numerical work is placed after the solution in words. In these problems the solution is, as I have said, a multiplication of the second kind, while the proof would be a multiplication of the first kind. To carry through the proof the Egyptian would have to multiply the given odd number by the different fractions of his answer and get the product as the sum of the partial products of these multiplications.[2] Take the case of 2 divided by 7. To prove that 1/4 1/28 times 7 equals 2 he would say:

1	7
1/2	3 1/2
\ 1/4	1 1/2 1/4
[3] 1/7	1
1/14	1/2
\ 1/28	1/4
Total	2.

[1] Outside of the table for the division of 2 by odd numbers, the word *seshemet* occurs nine times in the papyrus: in Problems 41, 42, 43, 44, 46, 58, 60, 65, 66. Peet himself always translates it "working out," and says that that is what it means in these nine problems (pages 22–23).

[2] See proofs of Problems 24–27, 30, 32–38. [3] See the solutions in Problems 24 and 34.

Here the sixth line is obtained by halving the preceding line in order to get the $\bar{28}$ of 7 called for by the answer that we are trying to prove, but in the solution (and in the papyrus) we have $\bar{4}$ as the fraction required to complete to 2 the 1 $\bar{2}$ $\bar{4}$ in line 3, and we get $\bar{28}$ as the multiplier necessary to produce it. That is, in the solution [1] we get the $\bar{28}$ from the $\bar{4}$, while in the proof we get the $\bar{4}$ from the $\bar{28}$. The only other way of carrying through the proof would be to interchange multiplier and multiplicand (see page 6). With this interchange the Egyptian would have said:

\1	$\bar{4}$ $\bar{28}$
\2	$\bar{2}$ $\bar{14}$
\4	1 $\bar{7}$
Total	2,

getting the total, if necessary, by applying his fractions to some number, say 28. This would be like the third step in the solutions of Problems 24–27.

In the latter part of the table the second step is omitted and only the result of the first step is given, but the third step is given for all of the numbers except 3, 35, and 91, and indicates clearly that we have, at least in each case except these, the solution and not the proof.

In the first step[2] of these solutions the author takes small multipliers

[1] In the cases of 2 divided by 17 and 19 the papyrus shows even more clearly than in the case 2 divided by 7 that the calculations given represent the solution and not the proof. See page 16.

[2] The problem worked out in this table has usually been described as the decomposition of a fraction with numerator 2 into a sum of fractions with numerator 1. Several eminent mathematicians have discussed it, notably Sylvester (1880) and Loria (1892). Others are listed in the Bibliography under Collignon, 1881. Of the discussions which I have seen the clearest is that by Loria. All of these writers, however, discuss the problem from the modern point of view. For example, a formula given as followed in some cases is $2/n = 1/a + 1/an$, where $a = (n + 1)/2$ (see Eisenlohr, 1877, pages 30–35). But no formula or rule has been discovered that will give all the results of the table, and Loria expressly says that he does not attempt to indicate how the old Egyptians obtained them.

Peet supposes that the Egyptian began by separating 2 into two parts, the first of which was greater than 1 and exactly contained in the given odd number. How to find this first part, or to know that it was contained in the given odd number, is really the thing that Peet claims that the papyrus does not tell us. However, he sees in the "reckoning" which he calls proof a clue to the method (page 36), his explanation being somewhat like mine. See also Bibliography under Neugebauer, 1926.

Hultsch declares positively that there is "no indication of the method," that the Egyptian reckoning "is indeed a study of mystery," and that "attempts to explain it . . . in spite of the darkness in which the ancient priestcraft has enveloped it, have hitherto not succeeded" (1895, page 5).

The way in which they worked out their problem is what has especially interested me.

until he gets a result a little less than 2. Most frequently he begins with $\bar{3}$ and then halves a sufficient number of times. Sometimes he halves without taking $\bar{3}$, and sometimes he takes $\frac{1}{10}$ or $\frac{1}{7}$. In some cases with these fractional multipliers he gets a whole number and can use the reciprocal of this for a multiplier and get the result more quickly. He always does this with a multiple of 3, except for 3 itself, 9, and 15, because $\bar{3}$ of it is a number whose reciprocal gives him at once $1\frac{1}{2}$. When the number is a multiple of 5 or 7, the fraction $\frac{1}{5}$ or $\frac{1}{7}$ gives him a whole number. In the Egyptian mind the case of $\bar{3}$ is similar to $\frac{1}{5}$ and $\frac{1}{7}$. Finally, a special method seems to have been employed for 35 and 91, and still another method for 101. These cases will be explained below.

In reproducing the table, I have marked the different cases A, B, AD, BD, C, and E; that is, I have used:

A when the author first takes $\bar{3}$;

B when he simply halves;

D along with A or B when he also uses $\frac{1}{10}$ or $\frac{1}{7}$;

C when at some step he gets a whole number and uses its reciprocal as a multiplier;

E for the three special cases of 35, 91, and 101.

In accordance with the foregoing rules I have marked each number with the letter indicating the kind of multiplier used, and it seems desirable to give the details of one problem of each case.

For A, I will take the case of 17, which is the first number for which the method is fully given. Beginning with the number itself the Egyptian writes:

1	17
$\bar{3}$	11 $\frac{1}{3}$
$\frac{1}{3}$	5 $\bar{3}$
$\frac{1}{6}$	2 $\frac{1}{2}$ $\frac{1}{3}$
\ $\frac{1}{12}$	1 $\frac{1}{4}$ $\frac{1}{6}$.

What remains to make 2 is the problem that constitutes the second step of this solution. This step is not explained in any of the examples that make up the table, but the method, as we have seen, is explained in Problems 21–23. In this case the amount needed is $\frac{1}{3}$ $\frac{1}{4}$.

There was, I think, no question of reducing or decomposing a fraction with numerator 2, for the Egyptians never had such a fraction to reduce. The author wishes to express, in the only way he knew, the quantities that, used as multipliers with these odd numbers, will produce 2. Nor do I believe that there was any mystery, or that the Egyptian mathematician tried to cover up his methods of reasoning. The reckoning given explains sufficiently his methods, and the methods indicated are the easiest for him to use.

To make up $\frac{1}{3}$ the author multiplies 17 by 3, getting 51, and, because 3 times 17 is 51, it follows that $\frac{1}{51}$ of 17 is $\frac{1}{3}$. In the same way, getting 68 as 4 times 17, he knows that $\frac{1}{68}$ of 17 is $\frac{1}{4}$. Thus his answer is $\frac{1}{12}$ $\frac{1}{51}$ $\frac{1}{68}$.

He has two ways of writing down this third step. There are two multiplications, the multiplier of the second being the reciprocal of the product of the first, and differing from it only in the dot used in writing the reciprocal of a whole number. Thus in getting the $\frac{1}{3}$ these multiplications are

3 times 17 is 51 and $\frac{1}{51}$ times 17 is $\frac{1}{3}$.

But he always omits one of these two numbers, either the multiplier of the second multiplication or the product of the first. Thus in this example he might write:

	3	51	$\frac{1}{3}$,
or else	3	$\frac{1}{51}$	$\frac{1}{3}$.

As it happens, the author uses the second form in this case, but in some cases he uses the first. It may be, however, that he always has in mind the first multiplication, even when he puts a dot over the product, and that the fraction written after this multiplication is put in simply to indicate its purpose.[1]

To illustrate B, I will take the case of 13. The papyrus says:

1	13
$\frac{1}{2}$	6 $\frac{1}{2}$
$\frac{1}{4}$	3 $\frac{1}{4}$
\ $\frac{1}{8}$	1 $\frac{1}{2}$ $\frac{1}{8}$.

There remains $\frac{1}{4}$ $\frac{1}{8}$ for which the multipliers are $\frac{1}{52}$ and $\frac{1}{104}$ and the answer is $\frac{1}{8}$ $\frac{1}{52}$ $\frac{1}{104}$.

An example of AD is the number 25. Here we have simply:

1	25	
\ $\frac{1}{15}$	1 $\frac{2}{3}$	
\ 3	[2]75	$\frac{1}{3}$.

This would seem to indicate that the author first took $\frac{1}{10}$, getting $2\frac{1}{2}$, and

[1] Gunn thinks (pages 127–128) that the second of these two forms should always have been used, and he maintains that many of the multiplications are illogically set out in the papyrus. He would always write the middle number in the first column as a fractional multiplier and the third number in the second column as product. It seems to me that the first of the two multiplications is the more important as the one that has to be worked out, while the second is written in the line at the top anyway.

[2] The papyrus says $\frac{1}{75}$.

then $\frac{2}{3}$, $\frac{2}{3}$ of $\frac{1}{10}$ being $\frac{1}{15}$. $\frac{2}{3}$ of 2 would be $1\frac{1}{3}$ and $\frac{2}{3}$ of $\frac{1}{2}$ would be $\frac{1}{3}$, making $1\frac{2}{3}$. Having obtained $1\frac{2}{3}$ he requires $\frac{1}{3}$ more to make 2 and the multiplier for this is $\frac{1}{15}$. His answer therefore is $\frac{1}{15}$ $\frac{1}{15}$.

An example of BD is the number 31. Here, supplying the first two lines, which are not given, we have:

1	31
$\frac{1}{10}$	3 $\frac{1}{10}$
\ $\frac{1}{20}$	1 $\frac{1}{2}$ $\frac{1}{20}$.

There remains $\frac{1}{4}$ $\frac{1}{5}$, for which the multipliers are obtained by the usual method, and his answer is $\frac{1}{20}$ $\frac{1}{124}$ $\frac{1}{155}$.

For C, I will take first the number 21, which is a multiple of 3. $\frac{2}{3}$ of 21 is 14; therefore $\frac{1}{14}$ of 21 is $1\frac{1}{2}$. There remains $\frac{1}{2}$; 2 times 21 is 42, so that $\frac{1}{42}$ of 21 is $\frac{1}{2}$, and the answer is $\frac{1}{14}$ $\frac{1}{42}$.

For a multiple of 5 take 65. We may suppose that the author proceded in this way:

	1	65
	$\frac{1}{10}$	6 $\frac{1}{2}$
	$\frac{1}{5}$	13
therefore	$\frac{1}{13}$	5
	$\frac{1}{26}$	2 $\frac{1}{2}$
	\ $\frac{1}{39}$	1 $\frac{2}{3}$.

There remains $\frac{1}{3}$; 3 times 65 is 195, and his answer is $\frac{1}{39}$ $\frac{1}{195}$. The last line of this multiplication is obtained by taking $\frac{2}{3}$ of the numbers in the preceding line, $\frac{2}{3}$ of $2\frac{1}{2}$ being the same as in the case of 25.

I have marked 35, 91, and 101 with E. These cases may be explained in the following way:

We have seen that the relations in this table may be regarded in two ways, either as expressing twice the reciprocal of an odd number, or as showing by what the odd number must be multiplied to make 2. In obtaining them the Egyptian nearly always takes the latter point of view, but in the case of 35 he seems to have considered that he was doubling the reciprocal of 35. To find the double of $\frac{1}{35}$ he applies this fraction to the number 210, which is 6 times 35. This is indicated by a 6 written in red under 35. The double of 6 is 12, equal to 7 and 5, and these numbers are $\frac{1}{30}$ and $\frac{1}{42}$ of 210. Therefore his answer is $\frac{1}{30}$ $\frac{1}{42}$. He writes out his answer in the usual form, but puts under each fraction the amount that it makes when applied to 210. Finally, in the lines below, where he usually puts the multiplications of his solution, he writes the two fractions of his answer with the results of multiplying

35 by them, namely, $1\frac{1}{6}$ and $\frac{2}{3}$ $\frac{1}{6}$. All multiplication work is omitted and what is written could be taken as indicating a proof, especially as the second product is not in a form that could have been obtained by the last step of the process employed in the other cases.[1]

For 91 the same process that he has used for 35 will lead to the form of result that he gives. By this process he would take $\frac{1}{91}$ as $\frac{1}{91}$ of 910, which is 10. Twice this is 20, equal to 13 and 7, and the fractions that give 13 and 7 are $\frac{1}{70}$ and $\frac{1}{130}$. Multiplying 91 by these two fractions gives him 1 $\frac{1}{3}$ $\frac{1}{10}$ and $\frac{2}{3}$ $\frac{1}{30}$, together making 2.

In the case of 101 he takes $\frac{1}{101}$ itself for his first fraction. This makes 1 and he has to find fractions to make up another 1, and so 2 in all. He thinks of the familiar combination $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$ and takes for the other fractions of his answer $\frac{1}{202}$, $\frac{1}{303}$, and $\frac{1}{606}$. This example is found entirely in the New York fragments. Eisenlohr and other writers before 1922 supposed that the table went only to 99.

In nearly all cases after 23, except multiples of 3, the author omits the successive multiplications by which he obtains the first fraction of his answer, and though we can tell what these multiplications were from the denominator of this fraction, it is not always easy to say in what order they were taken or just how they were carried through. We have to be guided largely by the form in which he gives the product produced by them. It may be worth while to make one or two suggestions as to the way in which he did this.

With the number 15 he begins "$\frac{1}{10}$ $1\frac{1}{2}$." He may have obtained this 10 by taking $\frac{2}{3}$ of 15 just as he takes $\frac{2}{3}$ of 21 and of each multiple of 3 after 21, but it is more probable that he notices that he can easily divide by 10, and that this division will give him a number between 1 and 2, and so he does not take $\frac{2}{3}$ of 15 at all. In each case of a multiple of 3 after 15, except 93, he mentions the fact that he takes $\frac{2}{3}$.

With 25 I have supposed that the author took first $\frac{1}{10}$ and then $\frac{2}{3}$. He may have taken $\frac{2}{3}$ first and then $\frac{1}{10}$. $\frac{2}{3}$ of 25 is $16\frac{2}{3}$. $\frac{2}{3}$ of 10 is $6\frac{2}{3}$, and therefore $\frac{1}{10}$ of $6\frac{2}{3}$ is $\frac{2}{3}$, and $\frac{1}{10}$ of $16\frac{2}{3}$ is $1\frac{2}{3}$. Similarly with 53, if he takes $\frac{2}{3}$ and then halves, getting $17\frac{2}{3}$, he will then say that $\frac{1}{10}$ of $16\frac{2}{3}$ is $1\frac{2}{3}$ and $\frac{1}{10}$ of 1 is $\frac{1}{10}$, making 1 $\frac{2}{3}$ $\frac{1}{10}$. That seems to be the easiest way to explain the result that he gives.

In the case of 43, if he takes $\frac{2}{3}$ and then proceeds to halve, he will get $28\frac{2}{3}$, $14\frac{1}{3}$, and $7\frac{1}{6}$, and the whole numbers suggest taking $\frac{1}{7}$ for a multiplier. There seems no other reason for his using 7 and taking $\frac{1}{42}$

[1] It may be, however, that the author was not thinking of a proof, but only attempted to put in something that would correspond to the usual form of solution.

as his first fraction. $\frac{1}{42}$ is so small that he has to get $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{7}$ to make 2, and in his answer he has four fractions. If he had halved twice more instead of multiplying by $\frac{1}{7}$ he would have had $\frac{1}{24}$ for his first fraction and one form for his final answer would have been $\frac{1}{24}$ $\frac{1}{344}$ $\frac{1}{516}$.

When the author takes $\frac{2}{3}$ and $\frac{1}{10}$ in the same example his result in some cases comes most simply by taking $\frac{2}{3}$ first and $\frac{1}{10}$ at the end. Examples are 47, 53, 79, and 89. Yet in other cases, as of 55, 73, 83, and 95, he apparently multiplied by $\frac{1}{10}$ first. In the case of 59 he takes $\frac{2}{3}$ twice. The most natural process of obtaining the form given seems to be:

1	59
$\frac{2}{3}$	39 $\frac{1}{3}$
$\frac{1}{3}$	19 $\frac{2}{3}$
$\frac{1}{6}$	9 $\frac{1}{2}$ $\frac{1}{3}$
$\frac{1}{12}$	4 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{6}$
$\frac{1}{18}$	3 $\frac{1}{6}$ $\frac{1}{9}$
$\frac{1}{36}$	1 $\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{18}$,

writing for $\frac{1}{6}$, 9 $\frac{1}{2}$ $\frac{1}{3}$ instead of 9 $\frac{2}{3}$ $\frac{1}{6}$, and halving again, so that his fractions will have even denominators, before he takes $\frac{2}{3}$ the second time.

The author uses the word *gem* meaning "find"[1] before the first denominator in the case of every number after 41 which is not a multiple of 3, except perhaps 101,[2] and also in the cases of 93 and 99, and he uses it before both denominators for 91. All of these cases, except perhaps 93 and 99, are the more difficult ones, and he does not put in the details but leaves them to the reader.

All of these various cases seem to indicate that there was no definite rule for determining the multipliers to be used, but probably the slow experience of different writers suggested different multipliers for different examples, as they seemed to them the easiest or gave results in the most satisfactory form.

In the table as here reproduced I have put: first, the letter or letters indicating the kind of multipliers employed; second, the number; third, the first fraction of the answer, this being the multiplier that produces a number a little less than 2; then the number a little less than 2 that is produced by this multiplier; the remainder necessary to make 2; and, finally, the answer.[3]

[1] Peet translates this, "found."

[2] The piece of the papyrus on which it would be written is still missing.

[3] This table is used in the following problems: 1–6, 30, 31, 33, 34, 36, 38, 42, 43, 67, 69, and 70, in some of them more than once; it is used backwards in 10, 17, 61, and 67.

	Number	First Multiplier	Corresponding Product	Remainder	Answer
A	3	2/3	2		2/3
A	5	1/3	1 2/3	1/3	1/3 1/15
B	7	1/4	1 1/2 1/4	1/4	1/4 1/28
A	9	1/6	1 1/2	1/2	1/6 1/18
A	11	1/6	1 2/3 1/6	1/6	1/6 1/66
B	13	1/8	1 1/2 1/8	1/4 1/8	1/8 1/52 1/104
BD	15	1/10	1 1/2	1/2	1/10 1/30
A	17	1/12	1 1/4 1/6 or 1 1/3 1/12	1/3 1/4	1/12 1/51 1/68
A	19	1/12	1 1/2 1/12	1/4 1/6	1/12 1/76 1/114
C	21	1/14	1 1/2	1/2	1/14 1/42
A	23	1/12	1 1/2 1/4 1/6 or 1 2/3 1/4	1/12	1/12 1/276
AD	25	1/15	1 2/3	1/3	1/15 1/75
C	27	1/18	1 1/2	1/2	1/18 1/54
A	29	1/24	1 1/6 1/24	1/2 1/8 1/6	1/24 1/58 1/174 1/232
BD	31	1/20	1 1/2 1/20	1/4 1/5	1/20 1/124 1/155
C	33	1/22	1 1/2	1/2	1/22 1/66
E	35	1/30 1/42	1 1/6 2/3 1/6		1/30 1/42
A	37	1/24	1 1/2 1/24	1/3 1/8	1/24 1/111 1/296
C	39	1/26	1 1/2	1/2	1/26 1/78
A	41	1/24	1 2/3 1/24	1/6 1/8	1/24 1/246 1/328
AD	43	1/42	1 1/42	1/2 1/3 1/7	1/42 1/86 1/129 1/301
C	45	1/30	1 1/2	1/2	1/30 1/90
AD	47	1/30	1 1/2 1/15	1/3 1/10	1/30 1/141 1/470
BD	49	1/28	1 1/2 1/4	1/4	1/28 1/196
C	51	1/34	1 1/2	1/2	1/34 1/102
AD	53	1/30	1 2/3 1/10	1/6 1/15	1/30 1/318 1/795
AD	55	1/30	1 2/3 1/6	1/6	1/30 1/330
C	57	1/38	1 1/2	1/2	1/38 1/114
AD	59	1/36	1 1/2 1/12 1/18	1/4 1/9	1/36 1/236 1/531
BD	61	1/40	1 1/2 1/40	1/4 1/8 1/10	1/40 1/244 1/488 1/610
C	63	1/42	1 1/2	1/2	1/42 1/126
C	65	1/39	1 2/3	1/3	1/39 1/195
BD	67	1/40	1 1/2 1/8 1/20	1/5 1/8	1/40 1/335 1/536
C	69	1/46	1 1/2	1/2	1/46 1/138
BD	71	1/40	1 1/2 1/4 1/40	1/8 1/10	1/40 1/568 1/710
AD	73	1/60	1 1/6 1/20	1/3 1/4 1/5	1/60 1/219 1/292 1/365

	Number	First Multiplier	Corresponding Product	Remainder	Answer
C	75	$\frac{1}{50}$	$1\ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{50}\ \frac{1}{150}$
C	77	$\frac{1}{44}$	$1\ \frac{1}{2}\ \frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{44}\ \frac{1}{308}$
AD	79	$\frac{1}{60}$	$1\ \frac{1}{4}\ \frac{1}{15}$	$\frac{1}{3}\ \frac{1}{4}\ \frac{1}{10}$	$\frac{1}{60}\ \frac{1}{237}\ \frac{1}{316}\ \frac{1}{790}$
C	81	$\frac{1}{54}$	$1\ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{54}\ \frac{1}{162}$
AD	83	$\frac{1}{60}$	$1\ \frac{1}{3}\ \frac{1}{20}$	$\frac{1}{4}\ \frac{1}{5}\ \frac{1}{6}$	$\frac{1}{60}\ \frac{1}{332}\ \frac{1}{415}\ \frac{1}{498}$
C	85	$\frac{1}{51}$	$1\ \frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{51}\ \frac{1}{255}$
C	87	$\frac{1}{58}$	$1\ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{58}\ \frac{1}{174}$
AD	89	$\frac{1}{60}$	$1\ \frac{1}{3}\ \frac{1}{10}\ \frac{1}{20}$	$\frac{1}{4}\ \frac{1}{6}\ \frac{1}{10}$	$\frac{1}{60}\ \frac{1}{356}\ \frac{1}{534}\ \frac{1}{890}$
E	91	$\frac{1}{70}$ $\frac{1}{130}$	$1\ \frac{1}{5}\ \frac{1}{10}$ $\frac{2}{3}\ \frac{1}{30}$		$\frac{1}{70}\ \frac{1}{130}$
C	93	$\frac{1}{62}$	$1\ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{62}\ \frac{1}{186}$
AD	95	$\frac{1}{60}$	$1\ \frac{1}{2}\ \frac{1}{12}$	$\frac{1}{4}\ \frac{1}{6}$	$\frac{1}{60}\ \frac{1}{380}\ \frac{1}{570}$
BD	97	$\frac{1}{56}$	$1\ \frac{1}{2}\ \frac{1}{8}\ \frac{1}{14}\ \frac{1}{28}$	$\frac{1}{7}\ \frac{1}{8}$	$\frac{1}{56}\ \frac{1}{679}\ \frac{1}{776}$
C	99	$\frac{1}{66}$	$1\ \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{66}\ \frac{1}{198}$
E	101	$\frac{1}{101}$	1	$\frac{1}{2}\ \frac{1}{3}\ \frac{1}{6}$	$\frac{1}{101}\ \frac{1}{202}\ \frac{1}{303}\ \frac{1}{606}$

Division of the Numbers 1–9 by 10

Table:

Number divided by 10	Result
1	$\frac{1}{10}$
2	$\frac{1}{5}$
3	$\frac{1}{5}\ \frac{1}{10}$
4	$\frac{1}{3}\ \frac{1}{15}$
5	$\frac{1}{2}$
6	$\frac{1}{2}\ \frac{1}{10}$
7	$\frac{2}{3}\ \frac{1}{30}$
8	$\frac{2}{3}\ \frac{1}{10}\ \frac{1}{30}$
9	$\frac{2}{3}\ \frac{1}{5}\ \frac{1}{30}$.

This table can be used by the author when he wishes to take $\frac{1}{10}$ of a number that is not a multiple of 10, just as the preceding table is used when he wishes to double the reciprocal of an odd number.[1]

The Rhind papyrus gives the table in full as above and then proves that it is correct for the numbers 1, 2, 6, 7, 8, and 9. Apparently the author intended to put in only the more difficult proofs. Thus he omits the proof for 5 divided by 10 because this is the same as $\frac{1}{2}$. He also omits the proof for 4 because 4 divided by 10 is the same as 2 divided by

[1] See, for example, Problem 30, proof.

5, which we have already in the first table. But it seems strange that he should omit the proof for 3 and put in those for 1 and 2. All that remains of the second proof in the Rhind papyrus would fit 2 or 3 equally well, and Eisenlohr assumes that it is for 3, but the New York fragments show that really it is for 2.

It is not difficult to show how the Egyptians would derive the expressions in this table. Take, for example, the case of 3. To divide 3 by 10 we have to multiply 10 so as to get 3. We have:

1	10
\ $\bar{10}$	1
\ $\bar{5}$	2
Total $\bar{5}$ $\bar{10}$;	

or, taking 7,

1	10
\ $\bar{\bar{3}}$	6 $\bar{\bar{3}}$
[1] $\bar{10}$	1
$\bar{15}$	$\bar{\bar{3}}$
\ $\bar{30}$	$\bar{3}$
Total $\bar{\bar{3}}$ $\bar{30}$.	

In order to make these problems seem practical the author supposes that we have a certain number of loaves of bread to divide among 10 men.

Multiplication by Certain Fractional Expressions

Problems 7–20 are problems in multiplication[2] that consist of two groups. In the first the multiplier is 1 $\bar{2}$ $\bar{4}$ and in the second, 1 $\bar{\bar{3}}$ $\bar{3}$. Problems 7 and 9–15 belong to the first group and problems 8 and 16–20 to the second. One problem of each group is given and then the rest of those of the first followed by the rest of those of the second. In Problem 7, $\bar{4}$ $\bar{28}$ is multiplied by 1 $\bar{2}$ $\bar{4}$ and in each of the problems of this group

[1] Instead of this he may have said, as in the first table, the remainder is $\bar{3}$; 3 times 10 is 30 and therefore $\bar{30}$ of 10 is $\bar{3}$.

[2] Before Problem 7, the author put in the papyrus the words "Example of making complete" and some writers have explained this by saying that the problem is to add to a given number certain fractional parts of it. Neugebauer (See Bibliography, 1926) regards these as completion problems for the division of 2 by 7 and 9 in the table at the beginning of the papyrus, because they include certain multiplications that arise in those two divisions. It seems to me more natural to regard them as simple multiplications and to suppose that the heading came to be put here in some way by mistake. Perhaps the author wrote down the heading for Problems 21–23 and then discovered that he had these multiplications to put in first.

the multiplicand is one of the numbers that can be obtained from $\frac{1}{4}$ $\frac{1}{28}$ by doubling or halving one or more times. In Problem 7, the multiplication is as follows:

1	$\frac{1}{4}$ $\frac{1}{28}$
$\frac{1}{2}$	$\frac{1}{8}$ $\frac{1}{56}$
$\frac{1}{4}$	$\frac{1}{16}$ $\frac{1}{112}$
Total	$\frac{1}{2}$.

$\frac{1}{4}$ $\frac{1}{28}$ is the number which we have found as the value of 2 divided by 7; and in some of the problems $\frac{1}{7}$ is given as $\frac{1}{2}$ of this number, $\frac{1}{14}$ as $\frac{1}{2}$ of $\frac{1}{7}$, and so on. In fact, some of these multiplications are carried through in both ways in different problems.[1]

In adding his partial products the Egyptian applies these fractional numbers to the number 28, using the method that I have already explained (page 7), and in Problems 7, 13, 14, and 15 he puts under each fraction the number that it makes when taken as a part of 28. Thus in Problem 7, $\frac{1}{4}$ and $\frac{1}{28}$ of 28 are 7 and 1, $\frac{1}{8}$ and $\frac{1}{56}$ are $3\frac{1}{2}$ and $\frac{1}{2}$, and $\frac{1}{16}$ and $\frac{1}{112}$ are 1 $\frac{1}{2}$ $\frac{1}{4}$ and $\frac{1}{4}$. These numbers added together give 14, which is $\frac{1}{2}$ of 28. Therefore the answer is $\frac{1}{2}$.

In Problem 15, $\frac{1}{128}$ and $\frac{1}{896}$ of 28 are $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ and $\frac{1}{32}$, and we have what may seem rather complicated quantities to add, even when we use 28, but they are simpler than the given quantities, being the reciprocals of powers of 2.

In the problems of the other group, 8 and 16–20, the author starts with a single fraction and multiplies by 1 $\frac{2}{3}$ $\frac{1}{3}$, so that the result is simply to double the fraction. The numbers taken are $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{1}{24}$. In this group the fractional numbers are applied to 18, and in the first and last two problems we find placed under the fractions their values as parts of this number. Nearly all of these parts are themselves fractions, but they are the reciprocals of powers of 2.

There are some interesting multiplications in these solutions. In Problem 17 the author has $\frac{2}{3}$ of $\frac{1}{3}$ equal to $\frac{1}{6}$ $\frac{1}{18}$. He may have put this down from memory or have copied it from some table; or he may have applied the rule given in Problem 61 (see below). Then for a half of $\frac{1}{6}$ $\frac{1}{18}$ he writes $\frac{1}{9}$ because his table for the division of 2 by odd numbers tells him that 2 times $\frac{1}{9}$ is $\frac{1}{6}$ $\frac{1}{18}$.

Problem 61 may also be considered in this connection as it consists of a multiplication table of various fractions. With 61 is associated

[1] It is interesting to notice that the multiplicands and partial products in these multiplications can be arranged in a table formed by taking $\frac{1}{2}$ $\frac{1}{14}$ as the first number and multiplying several times by $\frac{1}{2}$.

61B, which gives a rule for finding $\frac{2}{3}$ of a fraction.[1] The rule says, "To find $\frac{2}{3}$ of $\frac{1}{5}$ take thou its double and its six-fold, and do thou likewise for any fraction that may occur." In the table at the beginning of the papyrus for the division of 2 by odd numbers we notice that the quotient for 3 is simply $\frac{2}{3}$, but if the author had used the method employed for all multiples of 3 after 15 he would have obtained $\frac{1}{2}$ $\frac{1}{6}$, and he sometimes finds it convenient as here to use this expression in place of $\frac{2}{3}$. This rule is used a number of times in the arithmetical portions of the papyrus.[2] In Problem 33 it is employed with the fraction $\frac{1}{679}$.

It is to be noticed that in dealing with numbers which the modern mathematician calls fractions, the Egyptian regards the denominator as the important element, and therefore, although he wants to multiply the 5 by 2 and 6, he says, "Multiply $\frac{1}{5}$."

Division by a Fractional Number

Problems 24–38 are all essentially problems in division by a fractional expression. Problem 67 may also be included in this list. In other words, these are multiplication problems in which the product is one of the given numbers. But in all of them it is the multiplier and not the multiplicand that is the other given number. And therefore, as they stand, they cannot be solved directly by the Egyptian process of multiplication. We have seen (pages 6 and 7) that there are two methods of solving such problems. One method was to regard all of the numbers as mere numbers and change the problem so as to make the given multiplier a multiplicand and obtain the answer first as a multiplier. The other method was that of false position. The former method is used in Problems 30–34. The latter seems to be the method in all except these.

The first eleven of these problems (24–34) are sometimes called *'aha'*[3] or quantity problems because they nearly all begin with the word *'aha'* and use this word to denote the result of their calculations. In Problems 35, 37, and 38 the quantity is a measure of grain. In 36 the word for the measure is omitted and the problem by itself is a purely

[1] The word for fraction used here means literally "weak sign." Gunn (page 134) thinks that it is used for "uneven fraction." When the denominator of a fraction is an even number the rule is unnecessary; for all that is necessary is "to take $1\frac{1}{2}$ times this number and dot it."

[2] Namely, in Problems 17, 30, 32, 33, and for $\frac{2}{3}$ of $\frac{1}{3}$ in Problem 61.

[3] *'ḥ'*; this was formerly written *hau*.

numerical one, but it is written in the same phraseology save for the omission of this word and is solved by the same method, so that we can consider the four problems together.

Problem 24 is to find a quantity that with its $\frac{1}{7}$ will make 19. The author assumes 7, which with its $\frac{1}{7}$ makes 8, and then, to find the answer, he multiplies 7 by the number that multiplying 8 will give 19; 7, 8, and 19 as well as the answer all represent quantities of the same kind.

In each of the problems 24–27 the multiplier is 1 and a single fraction and he assumes as his answer the number that is the denominator of the fraction. In 28 and 29 he has two or three fractions and assumes the product of their denominators as his answer.

Problems 30–34 are far more complicated; nowhere, indeed, in the entire papyrus is more skill shown in dealing with long fractional expressions. In these problems it is required to determine what quantity a certain fractional number must multiply in order to produce a given product. But in solving them the Egyptian changes his point of view and determines what must multiply this fractional number.[1] In the proofs, however, he goes back to his original point of view and multiplies the answer by the given fractional number to get the number given as product.

In Problem 30 the author wishes to find what quantity $\frac{2}{3}$ $\frac{1}{10}$ must multiply in order to produce 10, and so changing his point of view as I have said, he proceeds to multiply $\frac{2}{3}$ $\frac{1}{10}$. He finds that multipliers amounting to 13 will give 9 and a series of fractions. Then we may suppose that he takes these fractions and the given expression as parts of 30. At any rate he finds that $\frac{1}{30}$ will just complete the fractions to 1, and that $\frac{1}{23}$ of the given expression will make this $\frac{1}{30}$. Thus his answer is $13\frac{1}{23}$.

In Problems 31 and 33 he has the same fractional expression, namely, 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{7}$,[2] the products to be obtained being 33 and 37. In both cases he multiplies this fractional expression in such a way as to get very nearly the product desired.

Thus to get 33 he doubles three times and halves twice, checking those multipliers that will give him very nearly this number. The numbers that he checks are equal to $14\frac{1}{4}$, and make up most of his answer. He adds the whole numbers and larger fractions of the corresponding

[1] See page 6.

[2] This is the same as 2 $\frac{1}{6}$ $\frac{1}{7}$. It is very unusual for him to write a fractional expression in which the fractional part is itself more than 1.

products, leaving for the present the smaller fractions, and he gets $32\frac{1}{2}$. The remaining $\frac{1}{2}$ required to make up all of 33 must then be equal to the smaller fractions of these products plus a further product by a multiplier yet to be determined. It remains only to get this multiplier. Choosing 42 as a convenient number, he applies his expressions to 42. The required $\frac{1}{2}$, taken as $\frac{1}{2}$ of 42, is equal to 21; the previously discarded fractions make $17\frac{1}{4}$ of it, and leave $3\ \frac{1}{2}\ \frac{1}{4}$ to be obtained by the multiplier that we are seeking. The original expression, $1\ \frac{2}{3}\ \frac{1}{2}\ \frac{1}{7}$, applied to 42, is equal to 97, and $\frac{1}{97}$ of it is 1; that is, $\frac{1}{97}$ as multiplier gives a product that, taken as a part of 42, equals 1. To get the product that equals $3\ \frac{1}{2}\ \frac{1}{4}$ he must take as multiplier[1] $3\ \frac{1}{2}\ \frac{1}{4}$ times $\frac{1}{97}$, and this multiplier, together with the $14\frac{1}{4}$ checked in the first multiplication, makes up his answer, which is

$$14\ \frac{1}{4}\ \frac{1}{56}\ \frac{1}{97}\ \frac{1}{194}\ \frac{1}{388}\ \frac{1}{679}\ \frac{1}{776}.$$

In Problem 33 to get nearly 37 he has only to carry his doubling one step further, 16 times $1\ \frac{2}{3}\ \frac{1}{2}\ \frac{1}{7}$ being equal to $36\ \frac{2}{3}\ \frac{1}{4}\ \frac{1}{28}$, or only $\frac{1}{21}$ less than 37. He applies his expressions to 42 to determine that this remainder is $\frac{1}{21}$ as well as to determine the multiplier that will produce it. To do the latter, since $\frac{1}{21}$ of 42 is 2, he has to use as multiplier 2 times $\frac{1}{97}$, and his answer is

$$16\ \frac{1}{56}\ \frac{1}{679}\ \frac{1}{776}.$$

In Problem 32 the multiplier is $1\ \frac{1}{3}\ \frac{1}{4}$ and the product to be obtained is 2, and in Problem 34 the multiplier is $1\ \frac{1}{2}\ \frac{1}{4}$ and the product to be obtained is 10. The latter may be associated with Problems 7 and 9–15.

In Problem 31 the answer, as we have seen, is particularly long, and the author makes no attempt at a proof, but in the other four problems he goes through the proof very carefully, so that it becomes more prominent than the solution.

In the proof of his answer to Problem 32 he multiplies $1\ \frac{1}{6}\ \frac{1}{12}\ \frac{1}{114}\ \frac{1}{228}$ by $1\ \frac{1}{3}\ \frac{1}{4}$. From the partial products he selects first the larger numbers, namely, 1, $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{4}$, or $1\ \frac{1}{2}\ \frac{1}{4}$, leaving $\frac{1}{4}$ to be made up from the remaining fractions, and by applying these to 912, the largest number whose reciprocal is found among them, he finds that their sum is exactly $\frac{1}{4}$, and that his answer is correct. In the same way he carries through the somewhat easier proof of Problem 34.

In the case of Problem 33 he multiplies his answer by $1\ \frac{2}{3}\ \frac{1}{2}\ \frac{1}{7}$. Adding the whole numbers and larger fractions of this multiplication,

[1] To find this multiplier he takes $\frac{1}{97}$ and its double and halves twice, getting the double from the table at the beginning of the papyrus.

that is, the parts that came from the whole number 16 of his answer, he has 36 $\bar{3}$ $\bar{4}$ $\overline{28}$, which is the same number that he got at the beginning as 16 times the given fractional number. The remaining fractions of his multiplication are so complicated that he seems to have some difficulty in carrying through his proof. In the first place, he applies all of his fractions to 5432, here again taking the largest number whose reciprocal is found among them, and he places under each of the three fractions of the sum already obtained, as well as under each of the fractions that he has not yet added, the number that it makes as a part of 5432. The numbers placed under $\bar{3}$, $\bar{4}$, and $\overline{28}$ make 5173$\bar{3}$. This subtracted from 5432 leaves 258$\bar{3}$, and the proof would be completed if he should show that this is the sum of the numbers placed under the fractions not yet added. Instead, he writes after $\bar{3}$ $\bar{4}$ $\overline{28}$ that there remains $\overline{28}$ $\overline{84}$ (although in the first part of the solution he had the simpler form $\overline{21}$ for this remainder), and that the two fractions of this remainder make 194 and 64$\bar{3}$ when taken as parts of 5432.

Problems 35–38 are numerically the same kind as the preceding. I have explained 35 on page 11 and the other three are solved in the same way.

In Problem 67 it is required to find how many cattle there are in a herd when $\bar{6}$ $\overline{18}$ of them makes 70, the number due as tribute to the owner. The numerical work of the solution is exactly like that for 35–38. We can suppose that the Egyptian, without going through all of the reasoning, recognized that it was a problem of the same kind, to be solved by the same process.

Separation in Given Proportions.[1]

Four of the miscellaneous problems, 62, 63, 65, and 68, are problems of separation in given proportions.

Problem 62 is about a bag with pieces of gold, silver, and lead in it. The meaning is not very clear, but the numerical problem is simply to divide 84 into three parts proportional to 12, 6, and 3. These numbers added together make 21, 84 is 4 times 21, and therefore the parts into which 84 is to be divided are 4 times the numbers 12, 6, and 3.

Problem 63 is exactly the same kind of problem and is solved in the same way. 700 loaves are to be divided among four men in the proportion of the numbers $\bar{3}$, $\bar{2}$, $\bar{3}$, and $\bar{4}$. The sum of these fractions is 1 $\bar{2}$ $\bar{4}$. Instead of dividing 700 by 1 $\bar{2}$ $\bar{4}$ the author divides 1 by this

[1] This is an important class of problems always considered in our arithmetics and algebras.

number and multiplies 700 by the result. 1 divided by 1 $\frac{1}{2}$ $\frac{1}{4}$ gives $\frac{1}{2}$ $\frac{1}{14}$, 700 times $\frac{1}{2}$ $\frac{1}{14}$ is 400, and 400 multiplied by the four given fractions gives the four parts into which 700 is to be divided, namely, 266$\frac{2}{3}$, 200, 133$\frac{1}{3}$, and 100.

In Problem 65, 100 loaves are to be divided among 10 men with three getting double portions, and in Problem 68, 100 *hekat* of grain are to be distributed to four overseers, the numbers of men in their gangs being 12, 8, 6, and 4, respectively. These are problems of the same kind and solved in the same way.

Other Arithmetical Problems

Problem 39 is also a division problem, but it is simply to divide 100 loaves among 10 men, 50 among 6, and 50 among 4.

In Problem 66 an amount of fat is given as required for a year, to find how much that is for a day. It is a simple division problem, solved by the multiplication of the divisor, but it seems to be regarded by the author as typical, so that at the end he tells us to do like this for any other problem of this kind.

Problems 69–78 have to do with the relative values of certain amounts of food or drink, the amount of grain in a unit of food or drink, and the amount of food or drink that can be made from a unit measure of grain. One of them, Problem 76, is particularly interesting as belonging to a class of problems that have been common in all of our arithmetics and algebras: problems in regard to the time that it takes to do a piece of work when men work together, or to fill a cistern when two or more pipes are running at the same time, or to row a certain distance up a stream and back—problems solved by first adding the reciprocals of the given numbers.[1]

Arithmetical and Geometrical Progressions

There are two problems, 40 and 64, that involve arithmetical progression, and one, 79, that involves geometrical progression. These are very interesting and seem to mark the acme of the Egyptians' skill in arithmetic.

Problem 40 is more interesting as illustrating false position, and I have explained it in connection with my discussion of that process (page 12).

[1] See D. E. Smith, 1923, volume 2, pages 536–541; also "On the Origin of Certain Typical Problems," *American Mathematical Monthly*, volume 24, pages 64–71. Problem 76 does not involve time, but it is of the same type.

In Problem 64, 10 *hekat* of barley are supposed to be divided among 10 men in such a way that the amounts received by the different men form an arithmetical progression with a common difference of **1/8**[1] of a *hekat*. Thus the terms, their sum, and the common difference are all concrete quantities, namely, certain amounts of barley, and the author is careful to use the special notation required for these quantities (the special forms of numbers of *hekat* and the "Horus eye" forms; see page 31), but the ordinary notation for numbers which represent the number of men, or differences, and for numbers used as multipliers. Thus there are given the number of terms, 10; the sum of the terms, 10 *hekat* of barley; and the common difference, **1/8** of a *hekat*. First he gets the mean or average, which is 1 *hekat*. Then to get the largest term he would add to this average the common difference one-half as many times as there are differences in all, but as the number of differences is an odd number he cleverly adds instead one-half of the common difference as many times as there are differences, that is, 9 times. Having obtained the amount of the largest portion, he gets the others by subtracting the common difference a sufficient number of times, and writes down the entire progression in descending order, each term as a certain amount of barley.

Problem 79 is a problem in which is calculated in two ways the sum of a geometrical progression. There are two columns. The first column indicates, it seems to me, a numerical method for determining the sum when the first term is equal to the ratio. This method may be stated in the following rule: In any geometrical progression whose first term is equal to the ratio, the sum of any number of terms is equal to the sum of one less number of terms plus 1 multiplied by the ratio. This rule the author follows in the first column, and in the second he performs the ordinary process of multiplication by the ratio and adds the terms together, getting the same result and thinking perhaps that he was proving the rule.[2]

[1] I am using black-faced type to represent the "Horus eye" forms. See page 31.

[2] Neugebauer (1926) constructs a table in which these two columns appear as the last column and row, and the calculations are all reduced to doublings and additions. See Bibliography.

EGYPTIAN MEASURES[1]

Measures of Capacity

The unit of volume or capacity, used especially in measuring grain, was the *hekat*, which can be determined as 292.24 cubic inches, or a little more than half a peck.[2] This was divided into 320 parts called *ro*, but the Egyptians also used as fractions of a *hekat* the fractions whose denominators are powers of 2 down to $\frac{1}{64}$, $\frac{1}{64}$ of a *hekat* being 5 *ro*. This series of fractions was peculiarly adapted to multiplication by doubling or halving. They were written in a special notation and have been called "Horus eye" fractions (see Bibliography under Möller, 1911).[3]

Besides using the "Horus eye" notation for parts of a *hekat*, the Egyptians had special hieratic signs for the numbers from 5 to 10 when used to express *hekat*. These signs seem to be ligatures of dots,[4] the sign for 10 being a long vertical stroke, representing perhaps ten dots one above another. The Egyptians had also a peculiar way of writing an expression for a large quantity of grain (but, so far as I know,

[1] This discussion is confined entirely to the subject as it is presented in the Rhind papyrus. A very elaborate study of Egyptian weights and measures was made by Griffith in 1892 (see Bibliography) and I have taken some statements from his article and checked many others by it, but for the most part my account is based on a study of the papyrus itself.

[2] See page 32. For some reason Peet calls the *hekat* a bushel, although he says that a *hinu* ($\frac{1}{10}$ of a *hekat*) was about 29.2 cubic inches (page 25). It is possible that he gets this word from Sethe, who uses the word *Scheffel* several times for *hekat* (1916, pages 74 and 80). Eisenlohr also uses the word *Scheffel*, but uses it for 10 *hekat*. Gunn calls the *hekat* a gallon (page 126), but the gallon is a liquid measure, at least in America, and for the same amount in dry measure it is better to say "half a peck," unless we use the word *hekat* itself. A note by G. P. G. Sobhy in *The Journal of Egyptian Archaeology*, volume 10, pages 283–284, gives the *hinu* in litres as determined by several vases which are inscribed with their values, but there is a considerable variation in the results from these different vases, the value of a *hinu* varying from .4028 to .544, making the value of a *hekat* vary from 4.028 to 5.44 litres. As the litre is 61.023 cubic inches, 292.24 cubic inches would be 4.789 litres.

[3] In the Rhind papyrus when some calculation gives a portion of a *hekat* expressed in ordinary fractions, these are reduced first to *ro*, and then to the "Horus eye" fractions. See Problems 35, 37, 38, 69, and 70. In 69 and 70, "Horus eye" fractions are changed to ordinary form before multiplication, as if for purposes of multiplication the author thought of them as mere numbers. Generally the multiplications are carried through with the "Horus eye" forms.

[4] In hieroglyphic these dots may have been written as little circles (see Möller, 1909, volume 1, page 66).

only in hieratic). The sign which represents a container of grain on its side, and forms a part of many words for different kinds of grain (see page 47) was also used in these expressions. When the amount was equal to or more than 100 *hekat*, this sign was written with the number of hundreds before it, and the signs for any smaller number of *hekat* next after it. Also 50 *hekat* and 25 *hekat* were put down as $\frac{1}{2}$ and $\frac{1}{4}$.[1] The number of whole *hekat* was followed by "Horus eye" fractions and by *ro* and fractions of a *ro*. In the case of 2, 3, or 4 *ro* the sign for the word *ro* was written under the number, while this sign without a number stood for 1 *ro*, and the fractions of a *ro* came after the sign.

Furthermore the Egyptians had not only the system of a simple *hekat* and its parts and multiples, but also systems of a double *hekat* and a quadruple *hekat* with their parts and multiples, each part or multiple of a double *hekat* being twice the corresponding part or multiple of a simple *hekat*, and each part or multiple of a quadruple *hekat* four times the corresponding part or multiple of a simple *hekat*; and they had the peculiar way of writing an expression for a large quantity of grain in the double and quadruple systems that they used for simple *hekat*. The double *hekat* was indicated by doubling the vertical sign in the word for *hekat* and the quadruple *hekat* by introducing an additional grain sign with four grains over it , but when these signs are not given we cannot always be sure that the system indicated is the simple system.[2] The quadruple *hekat* seems to have been called sometimes "a great *hekat*" (Griffith, volume 14, page 432), or "a great quadruple *hekat*" (Problem 68[3]).

We get information as to the size of these various measures, and, in particular, of the *hekat*, from Problems 41–46, where the capacities of certain granaries are computed from their dimensions. Here the unit of length is the cubit (*meh*), supposed to be the royal cubit, equal to 20.62 inches. The author states that $\frac{2}{3}$ of a cubed cubit is the *khar*, and then

[1] It looks as if the Egyptians thought of a hundred-*hekat* as a unit.

[2] Double *hekat* are indicated only in Problems 82 and 84. The quadruple system is used in the granary problems, Problems 41–46, in Problems 47 and 68, and in Number 86. When there is no indication of either the double or quadruple system, it may be possible sometimes to determine the system by the nature of the problem. In the *pefsu* problems (69–78) the number of loaves made from a *hekat* of material varies from 5 to 45. I should be inclined to think that even a simple *hekat* would make 5 rather large loaves. But in some of the problems the numbers may have been taken at random without thought of any particular application, and in some cases it may not have been the intention to restrict the problem to a particular system, it being equally good whether the *hekat* are simple, double or quadruple *hekat*.

[3] But not in Problems 41–47 as Peet implies (page 26).

that it takes 20 *khar* to make 100 quadruple *hekat* or 400 simple *hekat*. This would make the *hekat* 292.24 cubic inches, as stated above.

In addition to the *hekat* and all of its parts and multiples there was the *khar* just mentioned, and the *hînu*, equal to 1/10 of a *hekat*, although its origin was independent of the *hekat* system.

Measures of Area and Length

The only problems dealing with area are 48–55. The units of measure in these problems besides the cubit (page 32) are, first, the linear unit called *khet*[1] which is 100 royal cubits, and, second, the square *khet* called *setat*, which is 10,000 square cubits. The area of a field is expressed in terms of the *setat* and fractions of a *setat* in much the same way as the measure of a quantity of grain is expressed in terms of the *hekat* and fractions of a *hekat*. In the first place, the Egyptians used the fractions 1/2, 1/4, and 1/8 of a *setat* in the same way as they used the "Horus eye" fractions, and for these fractions also they had special forms.[2] Then smaller portions of a *setat* were expressed in terms of a unit that they called "cubit"[3] and seem to have thought of as a strip 1 *khet* or 100 cubits long and 1 cubit wide. I shall call this unit a "cubit-strip." 100 cubit-strips make a *setat*, and as **1/8** of a *setat* is equal to 12 1/2 cubit-strips, calculations with this system of units are not quite as simple as with the *hekat* system, where the smallest "Horus eye" fraction is equal to a whole number of *ro*. Thus in Problem 54 we have 1/5 of a *setat*, which is 20 cubit-strips, expressed as **1/8** *setat* 7 1/2 cubit-strips. The double of this is **1/4 1/8** *setat* 2 1/2 cubit-strips, and so on. Cubit-strips are denoted by placing an arm, the determinative and word-sign of the word *meh*, over the number, and *setat* by placing a little rectangle over the number, or, in hieratic, a heavy stroke or arc. In two cases (in Problem 48) this is written as a heavy dot. The special hieratic forms of 7 and 9 used for 7 and 9 *hekat*, are used also when these numbers express so many *setat*, and 10 *setat* are expressed by a heavy vertical stroke just as 10 *hekat* are expressed. (See Problems 48 and 53.) When there are more than 10 *setat* the number of 10's is expressed as an ordinary number as if a ten-*setat* was thought of as a unit. In fact, the

[1] Gardiner (1927, page 199) calls it a rod although it is more than 10 rods. Peet (page 24) compares it to our chain, which is 4 rods.

[2] I shall put these fractions, like the "Horus eye" fractions, in black-faced type.

[3] Or "cubit-of-land." This use of the word cubit in expressions of area is like our method of measuring cloth and other articles that are sold by the yard, or lots in a city that are sold by the front foot. See Griffith, volume 14, pages 410 ff., volume 15, page 306.

line used for 10 may be only the ordinary sign for 1 although in the case of *hekat* it is regarded as a ligature for ten dots or small circles. When there are ten-*setat* and unit *setat* the *setat* sign is placed over the number denoting units. Thus 36 *setat* in Problem 48 is written as an ordinary 3 and then a 6 with the *setat* sign over it, and 72 *setat* as an ordinary 7 and a 2 with the *setat* sign over it. Besides the peculiar signs for $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ *setat* we have a special sign for $\frac{1}{2}$ of a cubit-strip (Problems 54 and 55). Sometimes this is like the ordinary sign for $\frac{1}{8}$ and sometimes the same without the dot at the end.

In some of the area problems the scribe handles his units in a way that seems a little confusing. Thus to get the area of a rectangle he sometimes multiplies its length in cubits by its width in *khet*. This of course will give him the correct answer in cubit-strips.

The pyramid problems (56–60), which follow the problems on area, introduce a new unit of length called the palm or hand-breadth (*shesep*), which is $\frac{1}{7}$ of a cubit, and which is denoted sometimes by an arc over the number; and $\frac{1}{4}$ of a palm called a finger-breadth or finger.

EGYPTIAN GEOMETRY

The author of this papyrus was able to determine the areas of rectangles, triangles, and circles, and the volumes of cylinders and prisms, and he knew that in a right triangle the relation of the lengths of two sides determines one of the angles. In the papyrus he takes up problems of volume, then problems of area, and, finally, problems involving the relative lengths of the sides of a triangle.

Determination of Volumes

In the problems of volume, 41–46, the author calculates the amount of grain that can be stored in certain spaces or bins of given dimensions, and the dimensions of bins that will contain given amounts of grain. Rather strangely he takes up the case of cylinders first and then that of rectangular parallelepipeds. As to the latter, all that I need to say is that he multiplies the three dimensions together. Problem 44 is an example of the direct calculation of such a volume, the three dimensions being all equal to 10. Problem 45 is the inverse of 44, and 46 is a problem of the same kind as 45. It is to be noticed that the author takes first the inverse of a numerical problem that he has worked out directly, and therefore he knows the answer.[1] Problem 45 as stated has only the volume given. Taken as the inverse of 44 we should consider it as involving the taking of a cube root. What he does is to take two of the dimensions as each equal to 10, and to find the third. Problem 46 is not the inverse of a problem already solved, but here also he assumes two of the dimensions as each equal to 10 and gets the third dimension, which in this case is not the same number, but is one-third of it, or $3\frac{1}{3}$, so that the bin is a rectangular parallelepiped but not a cube.

In Problems 41–43 the author has given the dimensions of a cylindrical body to find the volume. Thus in 41 the diameter of the cylinder is given as 9 and the altitude as 10. In order to obtain the area of the base the author subtracts from the diameter its $\frac{1}{9}$, squares the remainder, and obtains 64 as the area of the circle. Multiplying this by 10 he gets 640 as the contents of the granary in cubed cubits.

Thus we see that in this problem he gives the relation between the

[1] In the same way 58 is the inverse of 57 and 59B is the inverse of 59.

area of a circle and its diameter. He does not explain how he discovered this relation, but from the fact that this problem in volume precedes 48, which states the relation for areas, it may be supposed that he determines the volume of a cylinder by some such process as the following:

He would quite possibly start with a cylinder of 9 units diameter because 9 with the Egyptians was a very important number and represented a group of the principal divinities.[1] He would then construct square prisms of the same height but of different bases, and he would find that the cylinder filled with water would almost exactly fill the prism whose side was 8 units. He was then able to judge that the base of the prism would be determined by subtracting from the diameter its $\frac{1}{9}$, and this gives the value of π as 3.1605. The fact that in this case a whole number for the side of the prism gives such a close approximation was probably a happy accident. In the next problem, having determined this relation, he takes for diameter 10, which does not give him a whole number for the side of the prism. This method, which we have already noted (page 8), of taking a simple numerical example and generalizing from that, is not from the mathematical point of view strictly legitimate and is liable to error unless the generalization is afterwards proved.

Determination of Areas

In Problem 48 is indicated the relation between a circle and its circumscribing square.[2]

In Problems 51–53 the Egyptian determines the area of a triangle by multiplying $\frac{1}{2}$ of its base, and the area of a trapezoid by multiplying $\frac{1}{2}$ of the sum of its bases, by the length of a line (*meret*) which, so far as our present knowledge goes, might be either the side or a line representing the altitude. In the latter case he would be correct. In case the triangle is isosceles with a narrow base as compared with its height, he would be nearly correct, even if *meret* means side. Personally I am inclined to think that this word does mean side in geometry, and that the author intended to consider only isosceles triangles with narrow bases. In Problem 51 the base is comparatively narrow, 4, with *meret* equal to 10.[3] In 53 it seems to be $4\frac{1}{2}$ with *meret* 14, while in 52 the

[1] Sethe (1916) page 38. There were a greater and a lesser "ennead," the personnel of which varied in different ages and localities.

[2] Such a figure occurs on a Babylonian tablet of 2000 B.C.; compare Gadd, 1922.

[3] The error is but little over 2%. In the other cases it is still smaller.

complete triangle has 6 for base and 60 for *meret*. This word in other connections means bank or wharf, which would indicate a side and not the altitude. It does not seem probable that the author had much conception of different kinds of triangles. We may suppose that he has in mind a piece of land, of a certain width at one end and coming to a point, or at least narrower, at the other end. Thus to get the area he thinks of a rectangle with the average width of the piece of land.[1]

Pyramids. The Relation of the Lengths of Two Sides of a Triangle

The relation of the lengths of two sides of a right triangle is illustrated in Problems 56–60, which deal with the distinguishing lines of a pyramid. In these problems the scribe uses certain special terms. In 56–59 he uses the words *ukha-thebet* and *per-em-us* for two lines, and "pyramid" for the structure. In Problem 60 he calls the structure *iwn* [2] and the two lines *sentet* and *kay-en-heru*, and the height is much greater in proportion to the base. In both cases he uses the word *seked*[3] for the relation of the lengths of the two lines, but he thinks of the *seked*, not as a ratio, but as so many palms per cubit.

The diagrams themselves do not show definitely what these lines are, and there are two opinions respecting them. Eisenlohr in his translation takes the *ukha-thebet* as the diagonal of the base and the *per-em-us* as the lateral edge, while he takes *sentet* and *kay-en-heru* as the side of the base and the altitude. Borchardt (1893) contends that *ukha-thebet* and *sentet* both mean the side of the base, and that *per-em-us* and *kay-en-heru* both mean altitude.[4]

[1] Sethe in his review of Peet (see Bibliography under Peet, 1923, 2) regards *meret* as "clearly" meaning height, because the 7 in 53 is written at the vertex of the triangle (but in 51 the 10 is written along the middle of the side), and because mention is made of only one *meret*. Peet suggests, "half doubtfully" as Sethe says, that *meret* probably means height, but his reasoning is rather inconclusive and not very clear.

Gunn suggests (page 133) that *meret* is a pair of lines forming a right angle with each other, one of them perpendicular to the base at one end, and the other passing through the vertex, or, with a truncated triangle, lying on the line opposite the base. This idea was suggested to him by a (vertical) cross-section of a wharf and the sloping bank under it, although he himself says that to the Egyptian mind triangles mostly lie flat on the ground. To me a bank or wharf suggests the dividing line between the water and the land and would be applied to the side of a triangle of land as separating the ground within from the ground outside. The idea of a cross-section of a wharf is one of four arguments that Gunn offers. The other three are not so definite and can be modified, if necessary, so as to apply equally well to the interpretation of *meret* as the side of the triangle.

[2] *yûn*, a word generally meaning "pillar."

[3] Peet translates this *batter*.

[4] This interpretation had been suggested by E. and V. Revillout (1881).

In the first interpretation the ratio involved in the *seked* in Problems 56–59B is the cosine of the angle which the lateral edge makes with the diagonal of the base, and in Problem 60 the tangent of the angle which the lateral face makes with the base. In the second interpretation the *seked* means the cotangent of the latter angle in all of the problems.[1]

Borchardt argues with considerable force from practical considerations and most Egyptologists have now accepted his interpretation, although it necessitates the assumption that the Egyptian called the same line by different names in successive problems.[2] In my Free Translation it will be convenient to translate these terms in accordance with Borchardt's theory.The whole matter is not very important from the point of view of Egyptian mathematics. The important point is that at the beginning of the 18th century B.C., and probably a thousand years earlier, when the greater pyramids were built, the Egyptian mathematician had some notion of referring a right triangle to a similar triangle, one of whose sides was a unit of measure, as a standard. Nor do the two interpretations make much difference in the angles of the structure. In fact, the actual measurements of the pyramids themselves vary so much that we cannot tell absolutely from measurement which is the more probable interpretation.

In 56, 58, 59, and 60, the lengths of the two lines are given to determine the *seked*; in 57 and 59B the base line and the *seked* are given to determine the other line. In all cases we have an isosceles triangle, a half of whose base is the base of the right triangle used.

[1] Assuming that in 60 the author divides the wrong way. See page 99.

[2] Borchardt suggests in another paper (1922, page 12) that Problem 60 may have been taken from a different source.

THE METHODS AND AIMS OF THE EGYPTIAN MATHEMATICIAN

His Methods

The Rhind mathematical papyrus is our chief authority for the state of mathematical knowledge in Egypt about the year 1650 B.C., and while some scholars such as E. and V. Revillout (1881) see in it only the work of a school-boy, most writers have recognized the scientific character of its procedure. The Egyptians of this date did not have the mental development reached by the Greeks a thousand years later. They had a smaller store of mathematical facts and less skill in mathematical operations. Yet their skill was remarkable and there was a scientific quality in their mathematics. The author of this papyrus had an idea of general methods applicable to groups of problems, and within the groups simple problems are followed by more complicated ones that are of the same kind and are solved by the same methods.

The methods of the Egyptian were largely those of trial and what we might call approximation. That is, if he could not get the answer at once he would try to get nearly the answer first and then make up what was lacking. This appears especially in what we have called the second kind of multiplication, where he has the multiplicand and product given to find the multiplier (see page 5), while the method of false position is also a method of trial.

Yet he was quick to generalize and when he had found a solution for a simple problem he did not hesitate to solve in the same way more difficult problems of the same kind, and occasionally to state the solution as a rule. He expresses this idea at the end of Problem 66, where he says, "Do the same thing in any example like this." Some processes are repeated again and again, showing that he had a method clearly in mind, even if he did not express it as a rule. One rule is put in words in Problem 61B, and is employed in several places (see pages 24–25) as if it were well understood. There are other operations and methods of solution that are taken for granted as being familiar at least to the author. His method of dividing a number in given proportions follows a definite rule which is employed without explanation, although not formulated (see page 28). Problem 79 illustrates, as I believe, a rule for finding the sum of a geometrical progression (see page 30).

In the geometrical problems he has several rules that he uses without comment, rules for getting areas and volumes and for changing from one unit of measure to another. Perhaps the most striking single operation that is often used is that by which in the third step of a multiplication of the second kind he determines the multiplier that will produce as product the reciprocal of a given number (see page 6).

A striking, though natural, characteristic of the Egyptian's work is his tendency noted above to take particular cases or particular numbers and generalize from them. This is seen especially in his method of adding fractions by taking them as parts of some number and adding their values when applied to this number (see pages 7–10). See also a remark on page 36.

It is quite possible that the writer of the papyrus, and the Egyptian mathematicians who preceded him, kept the results of their multiplications and other calculations in the form of tables, and often, when they had a multiplication or division to perform, took the result from these tables instead of working it out in full.[1] This would explain the fact that details are often omitted that are more difficult than other details that are put in. Some one must have worked out these details in some way, but the result of a multiplication once worked out could be used in two or three ways (see page 83). We have called the first part of the papyrus a table and next after this there is a table given in full just before Problem 1. There are also tables for fractions of a *hekat* in Problems 47, 80, and 81.

His Mistakes

Much has been written about the mistakes that we find in the Rhind papyrus. There are occasional mistakes, mostly numerical, that are merely accidental. Some of them are found in the group of Problems 7–20 (see page 64); five times in the first table 60 is written for 80; in two problems (43 and 59) the two given numbers are interchanged in the statement as compared with the solution; in Problem 49 the dimensions given in the statement are 10 and 2, but in the solution 10 and 1; in Problem 64 the mean share is given as ½ *hekat* when it is 1 *hekat*; in Problem 43 a second method of solution is preceded by one step of the first; in the last part of Problem 82 there is a numerical mistake in the division of a quantity of grain by 2. These are examples. At the beginning it is stated that this work is a copy of writings of an earlier period, and some of the mistakes seem to be mistakes of copying. Peet has

[1] We have many elaborate tables of the Babylonians. See, for example, Hilprecht, 1906.

explained very skillfully how they may have arisen (see pages 63, 119, and 124). There are indications in the papyrus of corrections and alterations made after it was completed.[1] We may think of an earlier author, or earlier authors, who wrote the original from which our copy was made; of the scribe A'h-mosè who may have been an ignorant copyist, but who understood something at least of the problems and may have added details of his own; corrections or changes in the copy that is now in the British Museum may have been made by a still later hand. It is not likely that A'h-mosè made all the mistakes and the earlier writers none.[2] In two or three problems there is some confusion as to the meaning of the problem or of portions of the solution (53, 84, and certain lines of 82), that may be due to the fact that some parts are missing or that the writer or some copyist attempted to put down a problem that was but imperfectly known to him. It is interesting to speculate on the relation of these different writers to the book, but for the most part we shall have to take it as we have it now, the product of Egyptian mathematics, and Egyptian mathematics at its highest development. As to the mistakes, if a problem is obscure it may be necessary to notice them, but if it is clear we are not interested in them. We wish to know what the Egyptians understood, what they could do, and the methods that they employed. Everyone makes mistakes, and almost any modern mathematical book, in spite of the careful proof-reading that it receives, contains mistakes just like those in the papyrus, and some of them contain many more than the papyrus. The remarkable thing is not that there are mistakes, but that most of the calculations are carried through without them.

There are some cases where the method employed does not give a result that is exactly correct. This indicates the limits of the Egyptian's knowledge and is a matter of interest. But we should hardly call the em-

[1] Thus in Problem 28, line 2, the number which should be 10 was originally written 20 (and it is so written in the British Museum Facsimile) but the little mark that would make the 20 appears somewhat vaguely, and, according to Peet, this is because it was imperfectly erased, so that the papyrus may now be regarded as saying 10. The reader will find that corrections were made in Problems 11 and 12 and that the form of expression was changed in certain lines of Problem 61 (see notes on Problem 61, and, in the Literal Translation, on Problems 11 and 12). Sometimes the correction was made in lighter ink. There are also many patches covering portions of the original writing that are copied on the patches. Some of these can be recognized in the photographs. An interesting example is the sign for *d* near the end of line 2 in Problem 67. This sign is made on a patch and a little bit of the end of the original sign can be seen projecting from under an edge of the patch.

[2] We do not know anything about A'h-mosè, but the mistakes are not all mistakes of copying and some of them show some understanding of the processes on the part of whoever made them.

ployment of such a method a careless mistake. He takes for the area of a circle the square of $\frac{8}{9}$ of the diameter, and according to one interpretation he seems to take for the area of a narrow isosceles triangle the product of the length of the side and the half of the base. In Problem 53 (according to my interpretation) he tries to allow for the deficiency of his figure (ABED, see page 94) from a rectangle by taking away its $\frac{1}{10}$ from the product of its base and side. We have some noteworthy indications of the limitations of the Egyptians, but we have also remarkable examples of what they could do, and in comparison the mere mistakes that we may find are of no importance or interest.

His Theoretical Interest

A careful study of the Rhind papyrus convinced me several years ago that this work is not a mere selection of practical problems especially useful to determine land values, and that the Egyptians were not a nation of shopkeepers, interested only in that which they could use. Rather I believe that they studied mathematics and other subjects for their own sakes. In the Rhind papyrus there are problems of area and problems of volume that might be of use to the farmer who owns land and raises grain. There are pyramid problems that might furnish specifications to the builders, or enable an interested observer to determine the dimensions of a pyramid before him. Many of the arithmetical problems concern a division of loaves or of a quantity of grain among a certain number of men, or the relative values of different amounts of food or drink. But when we come to examine the conditions laid down and the numbers involved in these various problems as well as the purely numerical ones, we see that they are more like theoretical problems put in concrete form. In one (Problem 63) 700 loaves are divided among four men in shares that are proportional to the four fractions $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, the first four terms of their fraction-series. In two (Problems 40 and 64) there is a dividing into shares that form an arithmetical progression, in Problem 67 the tribute for cattle is determined as $\frac{1}{6}$ $\frac{1}{18}$ of the herd and the problem asks for the number of the herd when the number of tribute cattle is given, and Problem 31 is a problem whose answer is

$$14\ \tfrac{1}{4}\ \tfrac{1}{56}\ \tfrac{1}{97}\ \tfrac{1}{194}\ \tfrac{1}{388}\ \tfrac{1}{679}\ \tfrac{1}{776}.$$

Such problems and such quantities were not likely to occur in the daily life of the Egyptians. Thus we can say that the Rhind papyrus, while

very useful to the Egyptian, was also "an example of the cultivation of mathematics as a pure science, even in its first beginnings."[1]

NOTE ON THE EGYPTIAN CALENDAR AND EGYPTIAN CHRONOLOGY—[2] There are three references in the Rhind papyrus to the Egyptian calendar. We find in the beginning the date when it was written, in Problem 66 the fact that 365 days make a year, and in Number 87 three dates and two of the days that were called epagomenal days.

The Egyptians counted their years from the beginning of the king's reign, starting anew with each new king, and, with the many breaks in their history and the uncertainty regarding many of their kings, it is difficult now to determine the dates of events, or the lengths of intervals between them.[3] In the Egyptian calendar year there was a certain shifting of the seasons and of the dates of events depending on them, that sometimes furnishes a clue for these determinations.

The Egyptian year consisted of 12 months of 30 days each and 5 days at the end that were regarded as not belonging to any month. The five days were called epagomenal days and were supposed to be the birthdays of five of the gods.[4] There were three seasons of four months each.

[1] It has been pointed out (Archibald, review of Peet, 1923, 2) that problems involving arithmetical and geometrical progressions seem to imply considerations not strictly practical and that the frustum of a pyramid problem of the Golenishchev papyrus (Turaev, 1917) is further evidence of the Egyptians' theoretical interest in science. Wieleitner (second review by him listed in the Bibliography under Peet, 1923, 2), after referring to this, says, "But is it not sufficient simply to refer to the *Hau*-reckoning, which, even if it refers to a measure (as do 35, 37, and 38, which belong with the '*hau*-reckoning' problems), is purely theoretical and in part complicated," and he ends with the words that I have quoted. See also Karpinski, 1917, page 258.

The scientific interest of the Egyptians is especially shown in the Edwin Smith papyrus, which contains careful and systematically arranged observations on the human body, and adds many interesting details, even in cases that are classified as hopeless, with no treatment recommended. See J. H. Breasted, "The Edwin Smith Papyrus," *The New York Historical Society, Quarterly Bulletin*, volume 6, 1922, pages 5–31.

[2] References: J. H. Breasted, *Ancient Records of Egypt*, Chicago, volume 1, 1906, pages 25–48; Eduard Meyer, "Ægyptische Chronologie," 1904, "Nachträge zur ægyptische Chronologie," 1907, *Abhandl. d. Berlin. Akad. d. Wiss.*, Berlin; F. K. Ginzel, *Handbuch der mathematischen und technischen Chronologie*, Leipzig, 1906, Band I, Einleitung und Kapitel II. The last mentioned book is very complete and thorough. A view of Egyptian chronology not at present generally accepted has been advocated by F. Petrie in "The length of Egyptian history," *British School of Archaeology in Egypt. Studies*, London, 1911, volume 2, pages 10–12; and another by A. Weigall in *A History of the Pharaohs*, New York, 1925.

[3] One very uncertain period lies between the twelfth and eighteenth dynasties, which included the time of the Hyksos kings and of the writing of our papyrus.

[4] Osiris, Horus, Set, Isis, and Nephthys. In Number 87 two of these birthdays are mentioned as belonging to the first month. This, however, is certainly a mistake, although Egyptologists have found it difficult to explain how the writer came to make it. See Peet page 130.

Names were given to the months, but each month was also known simply as the first, second, third, or fourth of the season to which it belonged. The most striking event of the Egyptian year is the rising of the Nile, which begins in July, and the seasons were named with reference to this event and its consequences. Thus the first season was called the season of the inundation (*akhet*, mentioned at the beginning of the papyrus and in Number 87), the second season was the season of the "going forth" (*prôt*) when the vegetation burst forth in the fields as the waters of the river were subsiding, and the third season was the season of summer (*shômu*, mentioned also in Number 87), when the earth became dry and parched before the coming of the next inundation.

These names show that at first the calendar year was made to begin about the time that the Nile began to rise. But the rising of the Nile depends on the solar year, which is about a quarter of a day longer than this Egyptian calendar year,[1] and so the Nile began to rise a day later after four years and a month later after 120 years, and after a time the entire season of the inundation came before the river began to rise, and was the dryest season of the three. They soon found that their year was a "wandering year," but continued to use it.

The beginning of the rise of the Nile, or any other event that depends on the seasons, is not as regular, nor as easy to determine exactly as some things that may be observed of the stars. The Egyptians early discovered that in the daily apparent revolution of the sun and stars around the earth the stars were continually gaining on the sun. If they watched a star, setting perhaps not long after sunset, they would notice that it set earlier each night than the night before, until at last it would disappear almost before there was darkness enough to make it visible. Then, after a certain number of days, they would see it in the east rising before the sun and appearing in the morning sky as a harbinger of the coming day. They noticed this, in particular, of Sirius, the brightest of all the fixed stars. The day when Sirius first appeared as a morning star had a religious significance. We can imagine the priests in the early morning, gazing out over the desert, waiting for the rising of the sun, and then one morning to the south of the place where the sun would appear some one would be the first to catch a glimpse of the rising star just as its light was being put out by the advancing sunlight. This was

[1] $365\frac{1}{4}$ days is called a Julian year. The solar year is a very little shorter. Newcomb gives its length in 1900 as 365.2422 days, with a yearly shortening of .0053 seconds, which would make the solar year about 365.2425 days in early Egyptian history (Ginzel, page 32). This is the same as $365\frac{97}{400}$, the number of days in our calendar year, there being 97 leap years in 400 years.

called the heliacal rising of Sirius, its rising with the sun, and this day marked the beginning of a kind of year that would end when the day came for the same event to occur again. This Sirius year, like the solar year, was a quarter of a day longer than the Egyptian calendar year, and its beginning, like the rise of the Nile, occurred one day later every four years.

The siderial year is longer than the solar year. The vernal equinox moves along the ecliptic at the rate of about 50″ a year, and when the sun reaches the vernal equinox it has still 50″ to go to reach the point where it passed the vernal equinox the year before.[1] If a star is on the ecliptic the time from the moment when the sun passes it to the moment when this event occurs again will be exactly a siderial year. We might suppose that the same would be true for any fixed star. If we think of the great circle of the ecliptic with the earth as a point at its center, we should say that the sun passes any fixed star when they have the same longitude, longitude being measured along the ecliptic. But the apparent daily revolution of the sun and stars around the earth is a motion parallel to the equator, and when the observer thinks of a star as overtaking and passing the sun, the passing that he thinks of is when they have the same right ascension, for right ascension is measured along the equator.

The heliacal rising of Sirius as observed by the Egyptians was not, indeed, the moment when Sirius passed the sun, even in this sense, for the star would have to rise before the upper edge of the sun in order to be visible. The refraction of the atmosphere and other conditions must be taken account of in determining this moment, and especially the fact that Sirius in the south reaches the horizon farther along on its circle of rotation than the sun. These conditions, however, for the most part, are the same from year to year and would not affect the length of time between two such events. But the rate of increase in the right ascension of the star varies with the position of the equator, and the star year is not usually the same as the siderial year. The difference is, however, a periodic one, with a period of over 25,000 years. During one part of this period the star year is longer than the siderial year, and during the other part it is shorter. It simply happens that during practically all of Egyptian history the Sirius year was almost exactly 365¼ days.[2] The

[1] The exact distance is 50.2564″ and it takes the sun 20 min. 23.8 sec., making the siderial year equal to 365.25636 days (Ginzel, pages 28 and 32).

[2] Let λ and β be the longitude and latitude of a star, ω the angle which the ecliptic makes with the equator, and γ the longitude of the point on the ecliptic which has the same right

Egyptians have left two or three records, each giving the day and month when Sirius rose with the sun in a certain year of some king's reign. The number of days difference in the calendar dates of two such records, multiplied by 4, will enable us to determine the number of years between the two events,[1] but as the shifting of a day in four years amounts to a year in 1460 years, it may be necessary to add 1460 or a multiple of this number to get the length of the entire interval.

NOTE ABOUT WORDS FOR GRAIN AND OTHER KINDS OF FOOD—There are ten or twelve words used in the papyrus to denote some kind of grain or food. As I have translated them they are:

1. Specific grains,

yôt	barley	used in Problem	64
bôdet	spelt	used in Problems	79, 82, and 86
sûet	wheat	used in Problem	82

also

yôt mehy	Lower Egyptian barley	used in Problem	83

ascension as the star. Then from the right triangles of the figure we can derive the equation

$$\tan \gamma = \tan \lambda - \frac{\tan \omega \tan \beta}{\cos \lambda}.$$

For a fixed star β is constant but λ increases at the rate of $50''$ in a calendar year. Then when $d\lambda$ is taken equal to $50''$, $d\gamma$ will be the increase of γ in a year and will measure the amount by which the star year exceeds the solar year.

A study of the above equation and of the equation for $d\gamma/d\lambda$ shows that λ and γ are together at $-\pi/2$ and $\pi/2$, and, if β is positive, then as they increase from one to the other of these values, γ increases more slowly than λ until it comes to the value 0, and then more rapidly. Beyond $\pi/2$ conditions are reversed, γ increasing at first more rapidly and afterwards more slowly.

If β is negative, that is, if the star is south of the ecliptic, as is the case with Sirius, conditions between $-\pi/2$ and $\pi/2$ are the same as the conditions above between $\pi/2$ and $3\pi/2$: as λ and γ increase, γ increases more rapidly than λ until it comes to the value 0, and then more slowly. During the period of Egyptian history γ was positive and increasing more slowly than λ, $d\gamma < d\lambda$, and the Sirius year was shorter than the siderial year. Ginzel gives a table (page 185) for the length of the Sirius year at different periods, showing that it was equal to the Julian year about 3231 B.C., and that by 139 A.D. it was 1 min. 29 sec. longer.

[1] At least to within less than 4 years. The following illustration shows how this fact is used (see Ginzel pages 194 and 200):

A record of the heliacal rising of Sirius on the 28th day of the month Epiphi (the third month of the season *shômu*) under Tehût-mosè III has been fixed as occurring in 1470 B.C. Now the Ebers papyrus records a rising as on the 9th of Epiphi in the 9th year of Amen-hotpe I. From the 9th to the 28th is 19 days which would give us an interval of 76 years. Therefore the Ebers rising occurred about 76 years earlier than that recorded under Tehût-mosè, that is, about 1546 B.C., which puts the beginning of Amen-hotpe's reign at about 1555 B.C. According to Breasted this date is 1557 B.C.

yôt shema'	Upper Egyptian barley	used in Problem	74
2. General term for grain,			
shes[1]	grain	used in Problems	35, etc.
3. Meal or flour,			
wedyet	*wedyet*-flour	used in Problems	72, etc.
noid	meal	used in Problems	69 and 70
4. Other foods,			
'ôd	fat	used in Problem	66
besha	*besha*[2]	used in Problem	71
heket[1]	beer	used in Problems	71, 77, and 78.

The grain and flour words as generally written contain the grain sign  (see page 32).

The word *shes* occurs about 16 times and is a general term for any kind of grain. Thus in Problem 82 it appears to refer to both spelt and wheat. In many places it is used to introduce the standard notation, explained on pages 31–32, with the "Horus eye" fractions, "It makes of grain" being almost equivalent to "Express in standard form."

Wedyet-flour is used in making bread and beer, and in Problem 82 in making feed for geese. Peet suggests that this may be a general term for flour or meal, just as *shes* is a general term for grain.

Peet gives a full and clear discussion of the uses of these words (pages 113–114).

[1] See notes to Problems 41 and 71 in the Literal Translation in regard to the readings of these words.

[2] A kind of grain or fruit, Erman and Grapow, *Wörterbuch der Aegyptischen Sprache*, Volume 1, Leipzig, 1926, page 478.

FREE TRANSLATION AND COMMENTARY[1]

Title, Date, and Scribe

Accurate reckoning. The entrance into the knowledge of all existing things and all obscure secrets. This book was copied in the year 33, in the fourth month of the inundation season, under the majesty of the king of Upper and Lower Egypt, 'A-user-Rê', endowed with life, in likeness to writings of old made in the time of the king of Upper and Lower Egypt, Ne-ma'et-Rê'. It is the scribe A'h-mosè who copies this writing.

[1] In the following pages I have endeavored to give in clear smooth English a free translation of the Rhind papyrus with some notes explaining details. In some places I have put in words or clauses that are omitted from the original, but found in other problems of the same group, or are required to express what I believe is the true meaning of the Egyptian. In some places where parts of a solution are misplaced I have arranged them in their proper order (see, for example, Problems 32 and 69). More particularly, I have corrected numerical mistakes and careless slips in the way of writing numbers. The reader will find the exact words of the original in the Literal Translation, and all corrections that have been made in the Free Translation may be found by a comparison of the two.

Notes giving explanations or interpretations of the text, except such as have been included in the Introduction, are placed in the following pages with the problems to which they refer. Notes relating to the Egyptian language or writing are placed with the Literal Translation.

CHAPTER I. EGYPTIAN ARITHMETIC

SECTION I

Table of the Division of 2 by Odd Numbers

2 divided by 3

Get 2 by operating on 3. $\frac{2}{3}$ of 3 is 2.

2 divided by 5

$\frac{1}{3}$ of 5 is 1 $\frac{2}{3}$, $\frac{1}{15}$ of 5 is $\frac{1}{3}$.
Working out:

1	5
$\frac{2}{3}$	3 $\frac{1}{3}$
[1] \ $\frac{1}{3}$	1 $\frac{2}{3}$
\ $\frac{1}{15}$	$\frac{1}{3}$.

2 divided by 7

$\frac{1}{4}$ of 7 is 1 $\frac{1}{2}$ $\frac{1}{4}$, $\frac{1}{28}$ of 7 is $\frac{1}{4}$.

1	7			
$\frac{1}{2}$	3 $\frac{1}{2}$		1	7
\ $\frac{1}{4}$	1 $\frac{1}{2}$ $\frac{1}{4}$		2	14
[2] \ 4	28	$\frac{1}{4}$	4	28.

2 divided by 9

$\frac{1}{6}$ of 9 is 1 $\frac{1}{2}$, $\frac{1}{18}$ of 9 is $\frac{1}{2}$.

1	9	
$\frac{2}{3}$	6	
$\frac{1}{3}$	3	
\ $\frac{1}{6}$	1 $\frac{1}{2}$	
\ 2	18	$\frac{1}{2}$.

[1] See Introduction, page 4. The use of these checks is somewhat arbitrary. They are sometimes put with the wrong numbers. Often they are used only in some of the multiplications of a problem. In Problems 7–20 there are no checks at all and in certain other problems also (45, 46, 49, 51, 56, 66, and 79) they are absent. In this Free Translation I have tried to be a little more uniform, but to follow the original approximately.

[2] See Introduction, page 17.

2 divided by 11

$\frac{1}{6}$ of 11 is 1 $\frac{2}{3}$ $\frac{1}{6}$, $\frac{1}{66}$ of 11 is $\frac{1}{6}$.

1	11			
$\frac{2}{3}$	7 $\frac{1}{3}$	1	11	
$\frac{1}{3}$	3 $\frac{2}{3}$	\ 2	22	
\ $\frac{1}{6}$	1 $\frac{2}{3}$ $\frac{1}{6}$	\ 4	44	
		Total 6	66	$\frac{1}{6}$.

2 divided by 13

$\frac{1}{8}$ of 13 is 1 $\frac{1}{2}$ $\frac{1}{8}$, $\frac{1}{52}$ of 13 is $\frac{1}{4}$, $\frac{1}{104}$ of 13 is $\frac{1}{8}$.

1	13	
$\frac{1}{2}$	6 $\frac{1}{2}$	
$\frac{1}{4}$	3 $\frac{1}{4}$	
\ $\frac{1}{8}$	1 $\frac{1}{2}$ $\frac{1}{8}$	
\ 4	52	$\frac{1}{4}$
\ 8	104	$\frac{1}{8}$.

2 divided by 15

$\frac{1}{10}$ of 15 is 1 $\frac{1}{2}$, $\frac{1}{30}$ of 15 is $\frac{1}{2}$.

1	15
\ $\frac{1}{10}$	1 $\frac{1}{2}$
\ $\frac{1}{30}$	$\frac{1}{2}$.

2 divided by 17

Get 2 by operating on 17. $\frac{1}{12}$ of 17 is 1 $\frac{1}{3}$ $\frac{1}{12}$, $\frac{1}{51}$ of 17 is $\frac{1}{3}$, $\frac{1}{68}$ of 17 is $\frac{1}{4}$.

Working out:

1	17			
$\frac{2}{3}$	11 $\frac{1}{3}$			
$\frac{1}{3}$	5 $\frac{2}{3}$	\ 1	17	
$\frac{1}{6}$	2 $\frac{1}{2}$ $\frac{1}{3}$	\ 2	34	
\ $\frac{1}{12}$	1 $\frac{1}{4}$ $\frac{1}{6}$	Total 3	51	$\frac{1}{3}$
Remainder	$\frac{1}{3}$ $\frac{1}{4}$	4	68	$\frac{1}{4}$.

In the fourth line of the multiplication the author might have said 2 $\frac{2}{3}$ $\frac{1}{6}$, and then the last line would have been 1 $\frac{1}{3}$ $\frac{1}{12}$, and he uses this form, in giving his answer above, for the part that $\frac{1}{12}$ of 17 makes towards 2. Somewhat similarly in the case of 23 he changes 1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{6}$ to 1 $\frac{2}{3}$ $\frac{1}{4}$.

2 divided by 19

$\frac{1}{12}$ of 19 is 1 $\frac{1}{2}$ $\frac{1}{12}$, $\frac{1}{76}$ of 19 is $\frac{1}{4}$, $\frac{1}{114}$ of 19 is $\frac{1}{6}$.

1	19				
$\frac{2}{3}$	12 $\frac{2}{3}$		1	19	
$\frac{1}{3}$	6 $\frac{1}{3}$		2	38	
$\frac{1}{6}$	3 $\frac{1}{6}$		4	76	$\frac{1}{4}$
\ $\frac{1}{12}$	1 $\frac{1}{2}$ $\frac{1}{12}$		Remainder	$\frac{1}{6}$	
Remainder	$\frac{1}{4}$ $\frac{1}{6}$				
			1	19	
			\ 2	38	
			\ 4	76	
		Total	6	114	$\frac{1}{6}$.

2 divided by 21

$\frac{1}{14}$ of 21 is 1 $\frac{1}{2}$, $\frac{1}{42}$ of 21 is $\frac{1}{2}$.

1	21	
\ $\frac{2}{3}$	14	1 $\frac{1}{2}$
\ 2	42	$\frac{1}{2}$.

2 divided by 23

$\frac{1}{12}$ of 23 is 1 $\frac{2}{3}$ $\frac{1}{4}$, $\frac{1}{276}$ of 23 is $\frac{1}{12}$.

1	23				
$\frac{2}{3}$	15 $\frac{1}{3}$		1	23	
$\frac{1}{3}$	7 $\frac{2}{3}$		\ 10	230	
$\frac{1}{6}$	3 $\frac{1}{2}$ $\frac{1}{3}$		\ 2	46	
\ $\frac{1}{12}$	1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{6}$	Total	12	276	$\frac{1}{12}$.
Remainder	$\frac{1}{12}$				

2 divided by 25

$\frac{1}{15}$ of 25 is 1 $\frac{2}{3}$, $\frac{1}{75}$ of 25 is $\frac{1}{3}$.

1	25	
\ $\frac{1}{15}$	1 $\frac{2}{3}$	
\ 3	75	$\frac{1}{3}$.

2 divided by 27

$\frac{1}{18}$ of 27 is 1 $\frac{1}{2}$, $\frac{1}{54}$ of 27 is $\frac{1}{2}$.

1	27	
\ $\frac{2}{3}$	18	1 $\frac{1}{2}$
\ 2	54	$\frac{1}{2}$.

2 divided by 29

Get 2 by operating on 29. $\frac{1}{24}$ of 29 is 1 $\frac{1}{6}$ $\frac{1}{24}$, $\frac{1}{58}$ of 29 is $\frac{1}{2}$, $\frac{1}{174}$ of 29 is $\frac{1}{6}$, $\frac{1}{232}$ of 29 is $\frac{1}{8}$.

Working out:

1	29	
\ $\frac{1}{24}$	1 $\frac{1}{6}$ $\frac{1}{24}$	
\ 2	58	$\frac{1}{2}$
\ 6	174	$\frac{1}{6}$
\ 8	232	$\frac{1}{8}$.

2 divided by 31

$\frac{1}{20}$ of 31 is 1 $\frac{1}{2}$ $\frac{1}{20}$, $\frac{1}{124}$ of 31 is $\frac{1}{4}$, $\frac{1}{155}$ of 31 is $\frac{1}{5}$.

1	31	
\ $\frac{1}{20}$	1 $\frac{1}{2}$ $\frac{1}{20}$	
\ 4	124	$\frac{1}{4}$
\ 5	155	$\frac{1}{5}$.

2 divided by 33

$\frac{1}{22}$ of 33 is 1 $\frac{1}{2}$, $\frac{1}{66}$ of 33 is $\frac{1}{2}$.

1	33	
\ $\frac{2}{3}$	22	1 $\frac{1}{2}$
\ 2	66	$\frac{1}{2}$.

2 divided by 35

35. $\frac{1}{30}$ of 35 is 1 $\frac{1}{6}$, $\frac{1}{42}$ of 35 is $\frac{2}{3}$ $\frac{1}{6}$.
6   7   5

2 times $\frac{1}{35}$ is $\frac{1}{30}$ $\frac{1}{42}$. For $\frac{1}{35}$ applied to 210 gives 6; and 2 times 6 is 12, or 7 and 5, which are $\frac{1}{30}$ and $\frac{1}{42}$ of 210.

1	35
\ $\frac{1}{30}$	1 $\frac{1}{6}$
\ $\frac{1}{42}$	$\frac{2}{3}$ $\frac{1}{6}$.

This example is explained in the Introduction, page 18. In the papyrus, following his usual practice when applying fractions to a particular number, the author writes 6, 7, and 5 under the 35, $\frac{1}{30}$, and $\frac{1}{42}$ of the first line.

2 divided by 37

$\bar{24}$ of 37 is 1 $\bar{2}$ $\bar{24}$, $\bar{111}$ of 37 is $\bar{3}$, $\bar{296}$ of 37 is $\bar{8}$.

1	37				
$\bar{\bar{3}}$	24 $\bar{\bar{3}}$		\1	37	
$\bar{3}$	12 $\bar{3}$		\2	74	
$\bar{6}$	6 $\bar{6}$	Total	3	111	$\bar{3}$
$\bar{12}$	3 $\bar{12}$		Remainder	$\bar{8}$	
\\$\bar{24}$	1 $\bar{2}$ $\bar{24}$		1	37	
Remainder	$\bar{3}$ $\bar{8}$		2	74	
			4	148	
			8	296	$\bar{8}$.

2 divided by 39

$\bar{26}$ of 39 is 1 $\bar{2}$, $\bar{78}$ of 39 is $\bar{2}$.

1	39	
\\$\bar{\bar{3}}$	26	1 $\bar{2}$
\2	78	$\bar{2}$.

2 divided by 41

Get 2 by operating on 41. $\bar{24}$ of 41 is 1 $\bar{\bar{3}}$ $\bar{24}$, $\bar{246}$ of 41 is $\bar{6}$, $\bar{328}$ of 41 is $\bar{8}$.

Working out:

1	41				
$\bar{\bar{3}}$	27 $\bar{3}$		1	41	
$\bar{3}$	13 $\bar{\bar{3}}$		\2	82	
$\bar{6}$	6 $\bar{\bar{3}}$ $\bar{6}$		\4	164	
$\bar{12}$	3 $\bar{3}$ $\bar{12}$	Total	6	246	$\bar{6}$
\\$\bar{24}$	1 $\bar{\bar{3}}$ $\bar{24}$		8	328	$\bar{8}$.
Remainder	$\bar{6}$ $\bar{8}$				

2 divided by 43

$\bar{42}$ of 43 is 1 $\bar{42}$, $\bar{86}$ of 43 is $\bar{2}$, $\bar{129}$ of 43 is $\bar{3}$, $\bar{301}$ of 43 is $\bar{7}$.

	1	43	
Find	\\$\bar{42}$	1 $\bar{42}$	
	\2	86	$\bar{2}$
	\3	129	$\bar{3}$
	\7	301	$\bar{7}$.

2 divided by 45

1/30 of 45 is 1 1/2, 1/90 of 45 is 1/2.

	1	45	
	\2/3	30	1 1/2
	\2	90	1/2.

2 divided by 47

1/30 of 47 is 1 1/2 1/15, 1/141 of 47 is 1/3, 1/470 of 47 is 1/10.

	1	47	
Find	1/30	1 1/2 1/15	
	\3	141	1/3
	\10	470	1/10.

2 divided by 49

1/28 of 49 is 1 1/2 1/4, 1/196 of 49 is 1/4.

	1	49	
Find	\1/28	1 1/2 1/4	
	\4	196	1/4.

2 divided by 51

1/34 of 51 is 1 1/2, 1/102 of 51 is 1/2.

	1	51	
	\2/3	34	1 1/2
	\2	102	1/2.

2 divided by 53

Get 2 by operating on 53. 1/30 of 53 is 1 2/3 1/10, 1/318 of 53 is 1/6, 1/795 of 53 is 1/15.

Working out:

	1	53					
Find	\1/30	1 2/3 1/10			1	53	
	\6	318	1/6		\10	530	
	Remainder	1/15			\5	265	
				Total	15	795	1/15.

2 divided by 55

$\frac{1}{30}$ of 55 is 1 $\frac{2}{3}$ $\frac{1}{6}$, $\frac{1}{330}$ of 55 is $\frac{1}{6}$.

	1	55	
Find	$\backslash\frac{1}{30}$	1 $\frac{2}{3}$ $\frac{1}{6}$	
	$\backslash 6$	330	$\frac{1}{6}$.

2 divided by 57

$\frac{1}{38}$ of 57 is 1 $\frac{1}{2}$, $\frac{1}{114}$ of 57 is $\frac{1}{2}$.

1	57	
$\backslash\frac{2}{3}$	38	1 $\frac{1}{2}$
$\backslash 2$	114	$\frac{1}{2}$.

2 divided by 59

$\frac{1}{36}$ of 59 is 1 $\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{18}$, $\frac{1}{236}$ of 59 is $\frac{1}{4}$, $\frac{1}{531}$ of 59 is $\frac{1}{9}$.

	1	59	
Find	$\backslash\frac{1}{36}$	1 $\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{18}$	
	$\backslash 4$	236	$\frac{1}{4}$
	$\backslash 9$	531	$\frac{1}{9}$.

2 divided by 61

$\frac{1}{40}$ of 61 is 1 $\frac{1}{2}$ $\frac{1}{40}$, $\frac{1}{244}$ of 61 is $\frac{1}{4}$, $\frac{1}{488}$ of 61 is $\frac{1}{8}$, $\frac{1}{610}$ of 61 is $\frac{1}{10}$.

	1	61	
Find	$\backslash\frac{1}{40}$	1 $\frac{1}{2}$ $\frac{1}{40}$	
	$\backslash 4$	244	$\frac{1}{4}$
	$\backslash 8$	488	$\frac{1}{8}$
	$\backslash 10$	610	$\frac{1}{10}$.

2 divided by 63

$\frac{1}{42}$ of 63 is 1 $\frac{1}{2}$, $\frac{1}{126}$ of 63 is $\frac{1}{2}$.

1	63	
$\backslash\frac{2}{3}$	42	1 $\frac{1}{2}$
$\backslash 2$	126	$\frac{1}{2}$.

2 divided by 65

Get 2 by operating on 65. $\frac{1}{39}$ of 65 is 1 $\frac{2}{3}$, $\frac{1}{195}$ of 65 is $\frac{1}{3}$.

Working out:

	1	65	
Find	$\backslash\frac{1}{39}$	1 $\frac{2}{3}$	
	$\backslash 3$	195	$\frac{1}{3}$.

2 divided by 67

$\frac{1}{40}$ of 67 is 1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{20}$, $\frac{1}{335}$ of 67 is $\frac{1}{5}$, $\frac{1}{536}$ of 67 is $\frac{1}{8}$.

	1	67	
Find	$\backslash\frac{1}{40}$	1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{20}$	
	$\backslash 5$	335	$\frac{1}{5}$
	$\backslash 8$	536	$\frac{1}{8}$.

2 divided by 69

$\frac{1}{46}$ of 69 is 1 $\frac{1}{2}$, $\frac{1}{138}$ of 69 is $\frac{1}{2}$.

1	69	
$\backslash\frac{2}{3}$	46	1 $\frac{1}{2}$
$\backslash 2$	138	$\frac{1}{2}$.

2 divided by 71

$\frac{1}{40}$ of 71 is 1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{40}$, $\frac{1}{568}$ of 71 is $\frac{1}{8}$, $\frac{1}{710}$ of 71 is $\frac{1}{10}$.

	1	71	
Find	$\backslash\frac{1}{40}$	1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{40}$	
	$\backslash 8$	568	$\frac{1}{8}$
	$\backslash 10$	710	$\frac{1}{10}$.

2 divided by 73

$\frac{1}{60}$ of 73 is 1 $\frac{1}{6}$ $\frac{1}{20}$, $\frac{1}{219}$ of 73 is $\frac{1}{3}$, $\frac{1}{292}$ of 73 is $\frac{1}{4}$, $\frac{1}{365}$ of 73 is $\frac{1}{5}$.

	1	73	
Find	$\backslash\frac{1}{60}$	1 $\frac{1}{6}$ $\frac{1}{20}$	
	$\backslash 3$	219	$\frac{1}{3}$
	$\backslash 4$	292	$\frac{1}{4}$
	$\backslash 5$	365	$\frac{1}{5}$.

2 divided by 75

$\frac{1}{50}$ of 75 is 1 $\frac{1}{2}$, $\frac{1}{150}$ of 75 is $\frac{1}{2}$.

1	75	
$\backslash\frac{2}{3}$	50	1 $\frac{1}{2}$
$\backslash 2$	150	$\frac{1}{2}$.

2 divided by 77

Get 2 by operating on 77. $\bar{44}$ of 77 is 1 $\bar{2}$ $\bar{4}$, $\bar{308}$ of 77 is $\bar{4}$.

Working out:

	1	77	
Find	\\$\bar{44}$	1 $\bar{2}$ $\bar{4}$	
	\\4	308	$\bar{4}$.

2 divided by 79

$\bar{60}$ of 79 is 1 $\bar{4}$ $\bar{15}$, $\bar{237}$ of 79 is $\bar{3}$, $\bar{316}$ of 79 is $\bar{4}$, $\bar{790}$ of 79 is $\bar{10}$.

	1	79	
Find	\\$\bar{60}$	1 $\bar{4}$ $\bar{15}$	
	\\3	237	$\bar{3}$
	\\4	316	$\bar{4}$
	\\10	790	$\bar{10}$.

2 divided by 81

$\bar{54}$ of 81 is 1 $\bar{2}$, $\bar{162}$ of 81 is $\bar{2}$.

1	81	
\\$\bar{\bar{3}}$	54	1 $\bar{2}$
\\2	162	$\bar{2}$.

2 divided by 83

$\bar{60}$ of 83 is 1 $\bar{3}$ $\bar{20}$, $\bar{332}$ of 83 is $\bar{4}$, $\bar{415}$ of 83 is $\bar{5}$, $\bar{498}$ of 83 is $\bar{6}$.

	1	83	
Find	\\$\bar{60}$	1 $\bar{3}$ $\bar{20}$	
	\\4	332	$\bar{4}$
	\\5	415	$\bar{5}$
	\\6	498	$\bar{6}$.

2 divided by 85

$\bar{51}$ of 85 is 1 $\bar{\bar{3}}$, $\bar{255}$ of 85 is $\bar{3}$.

	1	85	
Find	\\$\bar{51}$	1 $\bar{\bar{3}}$	
	\\3	255	$\bar{3}$.

2 divided by 87

$\frac{1}{58}$ of 87 is 1 $\frac{1}{2}$, $\frac{1}{174}$ of 87 is $\frac{1}{2}$.

1	87	
\\$\frac{2}{3}$	58	1 $\frac{1}{2}$
\\2	174	$\frac{1}{2}$.

2 divided by 89

Get 2 by operating on 89. $\frac{1}{60}$ of 89 is 1 $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{20}$, $\frac{1}{356}$ of 89 is $\frac{1}{4}$, $\frac{1}{534}$ of 89 is $\frac{1}{6}$, $\frac{1}{890}$ of 89 is $\frac{1}{10}$.

Working out:

1	89	
Find \\$\frac{1}{60}$	1 $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{20}$	
\\4	356	$\frac{1}{4}$
\\6	534	$\frac{1}{6}$
\\10	890	$\frac{1}{10}$.

2 divided by 91

$\frac{1}{70}$ of 91 is 1 $\frac{1}{5}$ $\frac{1}{10}$, $\frac{1}{130}$ of 91 is $\frac{2}{3}$ $\frac{1}{30}$.

1	91
Find \\$\frac{1}{70}$	1 $\frac{1}{5}$ $\frac{1}{10}$
Find \\$\frac{1}{130}$	$\frac{2}{3}$ $\frac{1}{30}$.

2 divided by 93

$\frac{1}{62}$ of 93 is 1 $\frac{1}{2}$, $\frac{1}{186}$ of 93 is $\frac{1}{2}$.

1	93	
\\$\frac{2}{3}$	62	1 $\frac{1}{2}$
\\2	186	$\frac{1}{2}$.

2 divided by 95

$\frac{1}{60}$ of 95 is 1 $\frac{1}{2}$ $\frac{1}{12}$, $\frac{1}{380}$ of 95 is $\frac{1}{4}$, $\frac{1}{570}$ of 95 is $\frac{1}{6}$.

1	95	
Find \\$\frac{1}{60}$	1 $\frac{1}{2}$ $\frac{1}{12}$	
\\4	380	$\frac{1}{4}$
\\6	570	$\frac{1}{6}$.

2 divided by 97

$\frac{1}{56}$ of 97 is 1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$, $\frac{1}{679}$ of 97 is $\frac{1}{7}$, $\frac{1}{776}$ of 97 is $\frac{1}{8}$.

	1	97	
Find	\\$\frac{1}{56}$	1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$	
	\\7	679	$\frac{1}{7}$
	\\8	776	$\frac{1}{8}$.

2 divided by 99

$\frac{1}{66}$ of 99 is 1 $\frac{1}{2}$, $\frac{1}{198}$ of 99 is $\frac{1}{2}$.

	1	99	
Find	\\$\frac{2}{3}$	66	1 $\frac{1}{2}$
	\\2	198	$\frac{1}{2}$.

2 divided by 101

Get 2 by operating on 101. $\frac{1}{101}$ of 101 is 1, $\frac{1}{202}$ of 101 is $\frac{1}{2}$, $\frac{1}{303}$ of 101 is $\frac{1}{3}$, $\frac{1}{606}$ of 101 is $\frac{1}{6}$.

Working out:

\\1	101	1
\\2	202	$\frac{1}{2}$
\\3	303	$\frac{1}{3}$
\\6	606	$\frac{1}{6}$.

SECTION II

PROBLEMS 1–6. TABLE OF THE DIVISION OF THE NUMBERS 1–9 BY 10

Table

1	divided	by	10	gives	$\frac{1}{10}$
2	"	"	"	"	$\frac{1}{5}$
3	"	"	"	"	$\frac{1}{5}$ $\frac{1}{10}$
4	"	"	"	"	$\frac{1}{3}$ $\frac{1}{15}$
5	"	"	"	"	$\frac{1}{2}$
6	"	"	"	"	$\frac{1}{2}$ $\frac{1}{10}$
7	"	"	"	"	$\frac{2}{3}$ $\frac{1}{30}$
8	"	"	"	"	$\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
9	"	"	"	"	$\frac{2}{3}$ $\frac{1}{5}$ $\frac{1}{30}$.

Problem 1

Example of dividing 1 *loaf among* 10 *men.*

Each man receives $\frac{1}{10}$.

Proof. Multiply $\frac{1}{10}$ by 10.

Do it thus:		
	1	$\frac{1}{10}$
	\2	$\frac{1}{5}$
	4	$\frac{1}{3}$ $\frac{1}{15}$
	\8	$\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$

Total 1 loaf, which is correct.

Problem 2

Divide 2 *loaves among* 10 *men.*

Each man receives $\frac{1}{5}$.

Proof. Multiply $\frac{1}{5}$ by 10.

Do it thus:		
	1	$\frac{1}{5}$
	\2	$\frac{1}{3}$ $\frac{1}{15}$
	4	$\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
	\8	1 $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{15}$

Total 2 loaves, which is correct.

Problem 3

Divide 6 *loaves among* 10 *men.*

Each man receives $\frac{1}{2}$ $\frac{1}{10}$.

Proof. Multiply $\frac{1}{2}$ $\frac{1}{10}$ by 10.

Do it thus:		
	1	$\frac{1}{2}$ $\frac{1}{10}$
	\2	1 $\frac{1}{5}$
	4	2 $\frac{1}{3}$ $\frac{1}{15}$
	\8	4 $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$

Total 6 loaves, which is correct.

Problem 4

Divide 7 *loaves among* 10 *men.*

Each man receives $\frac{2}{3}$ $\frac{1}{30}$.

Proof. Multiply $\frac{2}{3}$ $\frac{1}{30}$ by 10; the result is 7.

Do it thus:		
	1	$\frac{2}{3}$ $\frac{1}{30}$
	\2	1 $\frac{1}{3}$ $\frac{1}{15}$
	4	2 $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
	\8	5 $\frac{1}{2}$ $\frac{1}{10}$

Total 7 loaves, which is correct.

The answer in this case might have been written[1] $\frac{1}{2}$ $\frac{1}{5}$, but the Egyptian thinks of $\frac{2}{3}$ as the largest fraction that there is and prefers to use it wherever he can.

In this problem and the next two the double of $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$ is given as 1 $\frac{1}{2}$ $\frac{1}{10}$. Directly it is 1 $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{15}$. Perhaps these fractions were taken as parts of some number, say 30. They would make 18, or 15 and 3, which would be $\frac{1}{2}$ and $\frac{1}{10}$ of 30.

Problem 5

Divide 8 *loaves among* 10 *men.*

Each man receives $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$.

Proof. Multiply $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$ by 10; the result is 8.

Do it thus:	1	$\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
	\2	1 $\frac{1}{2}$ $\frac{1}{10}$
	4	3 $\frac{1}{5}$
	\8	6 $\frac{1}{3}$ $\frac{1}{15}$

Total 8 loaves, which is correct.

Problem 6

Divide 9 *loaves among* 10 *men.*

Each man receives $\frac{2}{3}$ $\frac{1}{5}$ $\frac{1}{30}$.

Proof. Multiply $\frac{2}{3}$ $\frac{1}{5}$ $\frac{1}{30}$ by 10.

Do it thus:	1	$\frac{2}{3}$ $\frac{1}{5}$ $\frac{1}{30}$
	\2	1 $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$
	4	3 $\frac{1}{2}$ $\frac{1}{10}$
	\8	7 $\frac{1}{5}$

Total 9 loaves, which is correct.

In the first doubling we have 1 $\frac{1}{3}$, $\frac{1}{3}$ $\frac{1}{15}$, and $\frac{1}{15}$. The two $\frac{1}{3}$'s make $\frac{2}{3}$ and the two $\frac{1}{15}$'s make $\frac{1}{10}$ $\frac{1}{30}$. Thus we get 1 $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$. The next doubling is the same as in Problem 4.

SECTION III

PROBLEMS 7–20. MULTIPLICATION BY CERTAIN FRACTIONAL EXPRESSIONS[2]

Problem 7

Multiply $\frac{1}{4}$ $\frac{1}{28}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{4}$ $\frac{1}{28}$ as parts of 28 these are	7	and 1
$\frac{1}{2}$	$\frac{1}{8}$ $\frac{1}{56}$ " " " " " "	3 $\frac{1}{2}$	" $\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{16}$ $\frac{1}{112}$ " " " " " "	1 $\frac{1}{2}$ $\frac{1}{4}$	" $\frac{1}{4}$
Total	$\frac{1}{2}$ since as a part of 28 this is 14.		

[1] Problem 54 is numerically the same problem, but the division is carried through there in such a way that $\frac{1}{2}$ $\frac{1}{5}$ is the form of the result.

[2] See Introduction, page 23.

Problem 7B

Multiply $\frac{1}{4}$ $\frac{1}{28}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{4}$ $\frac{1}{28}$	
$\frac{1}{2}$	$\frac{1}{8}$ $\frac{1}{56}$	
$\frac{1}{4}$	$\frac{1}{16}$ $\frac{1}{112}$	as parts of 28 these are 1 $\frac{1}{2}$ $\frac{1}{4}$ and $\frac{1}{4}$
Total	$\frac{1}{2}$.	

Problem 8

Multiply $\frac{1}{4}$ *by* 1 $\frac{2}{3}$ $\frac{1}{3}$.

1	$\frac{1}{4}$	as a part of 18 this is 4 $\frac{1}{2}$
$\frac{2}{3}$	$\frac{1}{6}$	" " " " " " " 3
$\frac{1}{3}$	$\frac{1}{12}$	" " " " " " " 1 $\frac{1}{2}$
Total	$\frac{1}{2}$ since	" " " " " " " 9.

Problem 9

Multiply $\frac{1}{2}$ $\frac{1}{14}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{2}$ $\frac{1}{14}$
$\frac{1}{2}$	$\frac{1}{4}$ $\frac{1}{28}$
$\frac{1}{4}$	$\frac{1}{8}$ $\frac{1}{56}$
Total	1.

Problem 10

Multiply $\frac{1}{4}$ $\frac{1}{28}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{4}$ $\frac{1}{28}$
$\frac{1}{2}$	$\frac{1}{7}$
$\frac{1}{4}$	$\frac{1}{14}$
Total	$\frac{1}{2}$.

Problem 11

Multiply $\frac{1}{7}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{7}$
$\frac{1}{2}$	$\frac{1}{14}$
$\frac{1}{4}$	$\frac{1}{28}$
Total	$\frac{1}{4}$.

Problem 12

Multiply $\frac{1}{14}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{14}$
$\frac{1}{2}$	$\frac{1}{28}$
$\frac{1}{4}$	$\frac{1}{56}$
Total	$\frac{1}{8}$.

Problem 13

Multiply $\frac{1}{16}$ $\frac{1}{112}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{16}$ $\frac{1}{112}$	as parts of 28 these are 1 $\frac{1}{2}$ $\frac{1}{4}$	and $\frac{1}{4}$
$\frac{1}{2}$	$\frac{1}{32}$ $\frac{1}{224}$	" " " " " " $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$	" $\frac{1}{8}$
$\frac{1}{4}$	$\frac{1}{64}$ $\frac{1}{448}$	" " " " " " $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$	" $\frac{1}{16}$
Total	$\frac{1}{8}$ since as a part of 28 this is 3 $\frac{1}{2}$.		

Problem 14

Multiply $\frac{1}{28}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{28}$	as a part of 28 this is 1
$\frac{1}{2}$	$\frac{1}{56}$	" " " " " " " $\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{112}$	" " " " " " " $\frac{1}{4}$
Total	$\frac{1}{16}$ since	" " " " " " "1 $\frac{1}{2}$ $\frac{1}{4}$.

Problem 15

Multiply $\frac{1}{32}$ $\frac{1}{224}$ *by* 1 $\frac{1}{2}$ $\frac{1}{4}$.

1	$\frac{1}{32}$ $\frac{1}{224}$	as parts of 28 these are $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$	and $\frac{1}{8}$
$\frac{1}{2}$	$\frac{1}{64}$ $\frac{1}{448}$	" " " " " " $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$	" $\frac{1}{16}$
$\frac{1}{4}$	$\frac{1}{128}$ $\frac{1}{896}$	" " " " " " $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$	" $\frac{1}{32}$
Total	$\frac{1}{16}$ since as a part of 28 this is 1 $\frac{1}{2}$ $\frac{1}{4}$.		

There are several mistakes in these multiplications in the original, and at the end of Problem 15 the author puts a sign that stands for the word "error." See notes to the Literal Translation.

Problem 16

Multiply $\frac{1}{2}$ *by* 1 $\frac{2}{3}$ $\frac{1}{3}$.

1	$\frac{1}{2}$
$\frac{2}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{6}$
Total	1.

Problem 17

Multiply 1/3 *by* 1 2/3 1/3.

1	1/3
2/3	1/6 1/18
1/3	1/9
Total	2/3.

Problem 18

Multiply 1/6 *by* 1 2/3 1/3.

1	1/6
2/3	1/9
1/3	1/18
Total	1/3.

Problem 19

Multiply 1/12 *by* 1 2/3 1/3.

1	1/12	as a part of 18 this is 1 1/2
2/3	1/18	" " " " " " " 1
1/3	1/36	" " " " " " " 1/2
Total	1/6 since	" " " " " " " 3.

Problem 20

Multiply 1/24 *by* 1 2/3 1/3.

1	1/24	as a part of 18 this is 1/2 1/4
2/3	1/36	" " " " " " " 1/2
1/3	1/72	" " " " " " " 1/4
Total	1/12 since	" " " " " " " 1 1/2.

SECTION IV

Problems 21–23. Problems in Completion

Problem 21

It is said to thee, Complete 2/3 1/15 *to* 1.

Applied to 15, 2/3 is 10 and 1/15 is 1, making 11; the remainder is 4. Multiply 15 so as to get 4.

1	15
1/10	1 1/2
\ 1/5	3
\ 1/15	1
Total	4.

Therefore $\bar{5}$ $\overline{15}$ is what is to be added to the given number.

For proof add them all together, namely,

$\bar{\bar{3}}$ $\bar{5}$ $\overline{15}$ $\overline{15}$, making 1;

for, applied to 15, these fractions are equal to

10 3 1 1, making 15.

After obtaining the remainder, 4, our author has to determine what fractions, taken as parts of 15, make 4. But this is the same as to say, What should multiply 15 to make 4—and so he takes $\overline{10}$, $\bar{5}$, and $\overline{15}$ of 15, and from the last two gets his result. The same process is used in the next problem, but in Problem 23 the fractions are given without explanation.

The solutions of this problem and the next are proved by adding together the fractions of the given expression and the fractions of the answer.

In the papyrus at the end of this solution are placed the words "Another, $\bar{5}$ $\overline{10}$ to be added." This has no connection with Problem 21, but the number $\bar{5}$ $\overline{10}$ is the answer to the problem that immediately follows.

Problem 22

Complete $\bar{\bar{3}}$ $\overline{30}$ *to* 1.

Applied to 30, $\bar{\bar{3}}$ $\overline{30}$ is 21. 30 exceeds 21 by 9. Multiply 30 so as to get 9.

1	30
\\ $\overline{10}$	3
\\ $\bar{5}$	6
Total	9.

Therefore $\bar{5}$ $\overline{10}$ is to be added to make the completion.

For proof add them all together, namely,

$\bar{\bar{3}}$ $\bar{5}$ $\overline{10}$ $\overline{30}$, making 1;

for, applied to 30, these fractions are equal to

20 6 3 1, making 30.

Problem 23

Complete $\bar{4}$ $\bar{8}$ $\overline{10}$ $\overline{30}$ $\overline{45}$ *to* $\bar{\bar{3}}$.

Applied to 45 these are equal to

11 $\bar{4}$ 5 $\bar{2}$ $\bar{8}$ 4 $\bar{2}$ 1 $\bar{2}$ 1,

which requires 6 $\bar{8}$ more to make up $\bar{\bar{3}}$ of 45, or 30. 6 $\bar{8}$ is equal to $\bar{9}$ $\overline{40}$ of 45. Therefore $\bar{9}$ $\overline{40}$ is to be added to the given number to make $\bar{\bar{3}}$.

For proof add them all together, namely,

$\bar{4}$ $\bar{8}$ $\bar{9}$ $\overline{10}$ $\overline{30}$ $\overline{40}$ $\overline{45}$,

and these with an additional $\frac{1}{8}$ make 1; for applied to 45 these fractions are equal to

11 $\frac{1}{4}$ 5 $\frac{1}{2}$ $\frac{1}{8}$ 5   4 $\frac{1}{2}$ 1 $\frac{1}{2}$ 1 $\frac{1}{8}$ 1   and 15.

The author does not explain how he gets the fractions $\frac{1}{9}$ $\frac{1}{40}$. Following the method of the first two problems of this section he would have to multiply 45 so as to get 6$\frac{1}{8}$. This he might have done as follows:

1	45
$\frac{1}{10}$	4 $\frac{1}{2}$
$\frac{1}{5}$	9
\\$\frac{1}{9}$	5
$\frac{1}{20}$	2 $\frac{1}{4}$
\\$\frac{1}{40}$	1 $\frac{1}{8}$
Total $\frac{1}{9}$ $\frac{1}{40}$.	

In the proof of this problem, where the number to be obtained is $\frac{2}{3}$, he adds in also a $\frac{1}{3}$ and so gets 1, which is itself the number to be obtained in Problems 21 and 22.

SECTION V

PROBLEMS 24–29. '*AHA*' OR QUANTITY PROBLEMS

Problem 24

A quantity and its $\frac{1}{7}$ *added together become* 19. *What is the quantity?*

Assume 7.

\\1	7
\\$\frac{1}{7}$	1
Total	8.

As many times as 8 must be multiplied to give 19, so many times 7 must be multiplied to give the required number.

1	8
\\2	16
$\frac{1}{2}$	4
\\$\frac{1}{4}$	2
\\$\frac{1}{8}$	1
Total 2 $\frac{1}{4}$ $\frac{1}{8}$.	
\\1	2 $\frac{1}{4}$ $\frac{1}{8}$
\\2	4 $\frac{1}{2}$ $\frac{1}{4}$
\\4	9 $\frac{1}{2}$

Do it thus: The quantity is	16 $\frac{1}{2}$ $\frac{1}{8}$,
$\frac{1}{7}$	2 $\frac{1}{4}$ $\frac{1}{8}$,
Total	19.

In the third multiplication, instead of multiplying 7 by 2 $\frac{1}{4}$ $\frac{1}{8}$, the author multiplies 2 $\frac{1}{4}$ $\frac{1}{8}$ by 7, this being easier. A similar change is made in each of the next three problems. See Introduction, page 6.

Problem 25

A quantity and its $\frac{1}{2}$ *added together become* 16. *What is the quantity?*

Assume 2.

\1	2
\\$\frac{1}{2}$	1
Total	3.

As many times as 3 must be multiplied to give 16, so many times 2 must be multiplied to give the required number.

\1	3
2	6
\4	12
$\frac{2}{3}$	2
\\$\frac{1}{3}$	1
Total 5 $\frac{1}{3}$.	
1	5 $\frac{1}{3}$
\2	10 $\frac{2}{3}$

Do it thus: The quantity is	10 $\frac{2}{3}$
$\frac{1}{2}$	5 $\frac{1}{3}$
Total	16.

Problem 26

A quantity and its $\frac{1}{4}$ *added together become* 15. *What is the quantity?*

Assume 4.

\1	4
\\$\frac{1}{4}$	1
Total	5.

As many times as 5 must be multiplied to give 15, so many times 4 must be multiplied to give the required number. Multiply 5 so as to get 15.

\1	5
\2	10
Total 3.	

Multiply 3 by 4.

1	3
2	6
\4	12;

The quantity is		12
	1/4	3
	Total	15.

Problem 27

A quantity and its 1/5 added together become 21. *What is the quantity?*

Assume 5.

\1	5
\1/5	1
Total	6.

As many times as 6 must be multiplied to give 21, so many times 5 must be multiplied to give the required number.

\1	6
\2	12
\1/2	3
Total 3 1/2.	

\1	3 1/2
2	7
\4	14.

The quantity is		17 1/2
	1/5	3 1/2
	Total	21.

Problem 28

A quantity and its 2/3 are added together and from the sum 1/3 of the sum is subtracted, and 10 *remains. What is the quantity?*

Subtract from 10 its 1/10, which is 1. The remainder is 9. This is the quantity; its 2/3, 6, added to 9, makes 15, and 1/3 of 15, taken away from 15, leaves 10. *Do it thus* . . . .

It may be supposed that our author first solved the problem as follows:

Assume 9.

\\1	9
\\$\frac{2}{3}$	6
Total	15
1	15
$\frac{1}{3}$	5
Remainder	10.

As many times as 10 must be multiplied to give 10, that is, once, so many times 9 must be multiplied to give the required number, and therefore the required number is 9. But now he notices that 9 is obtained by taking away its $\frac{1}{10}$ from 10, so he puts in the solution given in the papyrus.

The solution does not seem to be complete. The words, *Do it thus* ("The doing as it occurs") are usually put at the beginning of numerical work, and in no other problem are they at the end of the solution. Peet has suggested (page 63) that, in copying, the scribe came to these words and unconsciously let his eye pass to the same words in the next problem, the statement of the next problem and the beginning of its solution being also omitted. A similar omission occurs in the solution of Problem 70.

Problem 29

A quantity and its $\frac{2}{3}$ are added together, and $\frac{1}{3}$ of the sum is added; then $\frac{1}{3}$ of this sum is taken and the result is 10. What is the quantity?

	\\1	10
	\\$\frac{1}{4}$	2 $\frac{1}{2}$
	\\$\frac{1}{10}$	1
The quantity is		13 $\frac{1}{2}$
	$\frac{2}{3}$	9
	Total	22 $\frac{1}{2}$
	$\frac{1}{3}$	7 $\frac{1}{2}$
	Total	30
	$\frac{2}{3}$	20
	$\frac{1}{3}$	10.

As in the preceding problem it may be supposed that our author first solved the problem as follows:

Assume 27.

\\1	27
\\$\frac{2}{3}$	18
Total	45
$\frac{1}{3}$	15
Total	60
$\frac{2}{3}$	40
$\frac{1}{3}$	20.

As many times as 20 must be multiplied to give 10, so many times 27 must be multiplied to give the required number.

But at this point he seems to have changed the order of these numbers in his mind and to have said, As many times as 20 must be multiplied to give 27 so many times 10 must be multiplied to give the required number.

\\1	20
$\frac{1}{2}$	10
\\$\frac{1}{4}$	5
\\$\frac{1}{10}$	2

Total 1 $\frac{1}{4}$ $\frac{1}{10}$.

Therefore we must multiply 10 by 1 $\frac{1}{4}$ $\frac{1}{10}$ (see Peet, page 64).

SECTION VI

PROBLEMS 30–34. DIVISION BY A FRACTIONAL EXPRESSION [1]

Problem 30

If the scribe says, What is the quantity of which $\frac{2}{3}$ $\frac{1}{10}$ *will make* 10, *let him hear.*

Multiply $\frac{2}{3}$ $\frac{1}{10}$ so as to get 10.

\\1	$\frac{2}{3}$ $\frac{1}{10}$
2	1 $\frac{1}{3}$ $\frac{1}{5}$
\\4	3 $\frac{1}{15}$
\\8	6 $\frac{1}{10}$ $\frac{1}{30}$

Total 13. 13 times $\frac{2}{3}$ $\frac{1}{10}$ makes 9 and the fractions $\frac{2}{3}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{10}$, and $\frac{1}{30}$. The remainder is $\frac{1}{30}$. Take 30. $\frac{2}{3}$ $\frac{1}{10}$ of 30 is 23. Therefore $\frac{1}{30}$ of 30, or 1, will be $\frac{1}{23}$ of this. 13 $\frac{1}{23}$ is the required number.

For proof we multiply 13 $\frac{1}{23}$ by $\frac{2}{3}$ $\frac{1}{10}$.

1	13 $\frac{1}{23}$
\\$\frac{2}{3}$	8 $\frac{2}{3}$ $\frac{1}{46}$ $\frac{1}{138}$
\\$\frac{1}{10}$	1 $\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{230}$
Total	10.

To get the remainder $\frac{1}{30}$ the author could apply the fractions of his partial products, $\frac{2}{3}$, $\frac{1}{10}$, etc., to 30, their values taken this way making 29 in all, and requiring 1 part more to make the full 30, so that in order to make a full 10 he would require, in addition to what he already has, $\frac{1}{30}$.

In the multiplication of his proof we may notice that $\frac{2}{3}$ of 13 is given at once as 8 $\frac{2}{3}$ and that $\frac{2}{3}$ of $\frac{1}{23}$ is given by the rule in Problem 61 (see Introduction, page 25).

[1] Full explanations of these problems are given in the Introduction, pages 25–28. I may mention that the problems of this section and the next are numbered by Eisenlohr a little differently from the order in the papyrus, 33 and 34 coming, in fact, after 38, but belonging in the same group with 30, 31, and 32. As I have used his numbering, I have followed his order.

$\frac{1}{10}$ also he can take without difficulty. For 13 he has $\frac{1}{10}$ of 10 equal to 1, and $\frac{1}{10}$ of 3, by the table that precedes Problem 1, is $\frac{1}{5}$ $\frac{1}{10}$. Finally, $\frac{1}{10}$ of $\frac{1}{23}$ is $\frac{1}{230}$.

In the original our author seems to say that we must multiply $\frac{1}{30}$ by $\frac{1}{23}$ to get $\frac{2}{3}$ $\frac{1}{10}$. It may be that he did not mean to put a dot (the sign for fraction) over 23, but meant to say, Multiply $\frac{1}{30}$ by 23 to get $\frac{2}{3}$ $\frac{1}{10}$. This is a correct statement but does not explain his solution. Following the method given in the other problems of this group, he should have said, Multiply $\frac{1}{23}$ by $\frac{2}{3}$ $\frac{1}{10}$ to get $\frac{1}{30}$.

Problem 31

A quantity, its $\frac{2}{3}$, *its* $\frac{1}{2}$, *and its* $\frac{1}{7}$, *added together, become* 33. *What is the quantity?*

Multiply 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{7}$ so as to get 33.

1	1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{7}$
\2	4 $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{28}$
\4	9 $\frac{1}{6}$ $\frac{1}{14}$
\8	18 $\frac{1}{3}$ $\frac{1}{7}$
$\frac{1}{2}$	$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{14}$
\\$\frac{1}{4}$	$\frac{1}{4}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{1}{28}$

Total 14 $\frac{1}{4}$. 14 $\frac{1}{4}$ times 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{7}$ makes 32 $\frac{1}{2}$ plus the small fractions $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$ $\frac{1}{28}$. 32 $\frac{1}{2}$ from 33 leaves the remainder $\frac{1}{2}$ to be made up by these fractions and a further product by a number yet to be determined.

$\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{14}$ $\frac{1}{28}$ $\frac{1}{28}$

taken as parts of 42 are

6 5$\frac{1}{4}$ 3 1 $\frac{1}{2}$ 1 $\frac{1}{2}$,

making in all 17 $\frac{1}{4}$, and requiring 3 $\frac{1}{2}$ $\frac{1}{4}$ more to make 21, $\frac{1}{2}$ of 42.

Take 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{7}$ as applying to 42:

\1	42
\\$\frac{2}{3}$	28
\\$\frac{1}{2}$	21
\\$\frac{1}{7}$	6
Total	97.

That is, 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{7}$ applied to 42 gives 97 in all. $\frac{1}{42}$ of 42, or 1, will be $\frac{1}{97}$ of this, and 3 $\frac{1}{2}$ $\frac{1}{4}$ will be 3 $\frac{1}{2}$ $\frac{1}{4}$ times as much. Therefore we multiply $\frac{1}{97}$ by 3 $\frac{1}{2}$ $\frac{1}{4}$.

[1]\\$\frac{1}{97}$	$\frac{1}{42}$	or 1	as a part of	42
\\$\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$	$\frac{1}{21}$	" 2	" " "	" 42
\\$\frac{1}{194}$	$\frac{1}{84}$	" $\frac{1}{2}$	" " "	" 42
\\$\frac{1}{388}$	$\frac{1}{168}$	" $\frac{1}{4}$	" " "	" 42.

[1] These multiplications do not represent the multiplication of $\frac{1}{97}$ by 3 $\frac{1}{2}$ $\frac{1}{4}$, but form a continuation of the multiplication at the beginning of the solution. But the partial products of

The total is 14 $\frac{1}{4}$ $\frac{1}{56}$ $\frac{1}{97}$ $\frac{1}{194}$ $\frac{1}{388}$ $\frac{1}{679}$ $\frac{1}{776}$, which multiplied by 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{7}$ makes 33.

Problem 32

A quantity, its $\frac{1}{3}$, *and its* $\frac{1}{4}$, *added together, become* 2. *What is the quantity?*

Multiply 1 $\frac{1}{3}$ $\frac{1}{4}$ so as to get 2.

1	1 $\frac{1}{3}$ $\frac{1}{4}$
\ $\frac{2}{3}$	1 $\frac{1}{18}$
\ $\frac{1}{3}$	$\frac{1}{2}$ $\frac{1}{36}$
\ $\frac{1}{6}$	$\frac{1}{4}$ $\frac{1}{72}$
\ $\frac{1}{12}$	$\frac{1}{8}$ $\frac{1}{144}$.

Take 12 times 12.

1	12
2	24
\4	48
\8	96
Total	144.

We will apply our fractions to 144. For the given expression we have

\1	144
\ $\frac{1}{3}$	48
\ $\frac{1}{4}$	36
Total	228.

The products above, taken as parts of 144, are equal to

228 152 76 38 19.

The sum of the numbers here that correspond to the multipliers checked is equal to 285 and requires 3 more to make up 288, or 2 times 144. As 1 $\frac{1}{3}$ $\frac{1}{4}$ times 144 is 228 we shall have as a continuation of our first multiplication,

\ $\frac{1}{228}$	$\frac{1}{144}$ or 1 as a part of 144
\ $\frac{1}{114}$	$\frac{1}{72}$ " 2 " " " " 144.

Adding together all the multipliers checked in this multiplication, we have 1 $\frac{1}{6}$ $\frac{1}{12}$ $\frac{1}{114}$ $\frac{1}{228}$ as the required quantity.

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the former multiplication are the multipliers of the latter and so are represented in the left-hand column here, while the multipliers of the former are the alternative numbers, 1, 2, $\frac{1}{2}$, and $\frac{1}{4}$, given at the right.

Proof.

\ 1	1 $\bar{6}$ $\bar{12}$ $\bar{114}$ $\bar{228}$
$\bar{\bar{3}}$	$\bar{\bar{3}}$ $\bar{9}$ $\bar{18}$ $\bar{171}$ $\bar{342}$
\ $\bar{3}$	$\bar{3}$ $\bar{18}$ $\bar{36}$ $\bar{342}$ $\bar{684}$
$\bar{2}$	$\bar{2}$ $\bar{12}$ $\bar{24}$ $\bar{228}$ $\bar{456}$
\ $\bar{4}$	$\bar{4}$ $\bar{24}$ $\bar{48}$ $\bar{456}$ $\bar{912}$.

The total is 1 $\bar{2}$ $\bar{4}$ and a series of smaller fractions. 1 $\bar{2}$ $\bar{4}$ taken from 2 leaves a remainder of $\bar{4}$. Apply the smaller fractions to 912. The fractions are:

$\bar{12}$ $\bar{114}$ $\bar{228}$ $\bar{18}$ $\bar{36}$ $\bar{342}$ $\bar{684}$ $\bar{24}$ $\bar{48}$ $\bar{456}$ $\bar{912}$,

and as parts of 912 they are equal to

76 8 4 50 $\bar{\bar{3}}$ 25 $\bar{3}$ 2 $\bar{\bar{3}}$ 1 $\bar{3}$ 38 19 2 1,

which together make 228, or $\bar{4}$ of 912. For

1	912
$\bar{2}$	456
$\bar{4}$	228.

In this problem the fractional number is 1 $\bar{2}$ $\bar{4}$ and the product to be obtained is 2. In order to add the partial products when multiplying 1 $\bar{2}$ $\bar{4}$ so as to get 2 the author applies the fractions to 144, getting this number apparently as 12 times 12, although he might have taken it as the largest number whose reciprocal occurs among the fractions to be added, which is his usual procedure.

In his first multiplication he finds that the multiplier 1 $\bar{6}$ $\bar{12}$ gives him very nearly 2. It may be noticed that he checks the $\bar{\bar{3}}$ and $\bar{3}$ instead of the first line. To produce exactly 2 he has to take as additional multipliers $\bar{228}$ and $\bar{114}$, getting $\bar{228}$ by the same process of reasoning that gave him $\bar{67}$ in the preceding problem and $\bar{53}$ in Problem 30.

He gives the answer as 1 $\bar{6}$ $\bar{12}$ $\bar{114}$ $\bar{228}$ and proceeds to prove that it is correct. Going back to his original point of view, he multiplies it by the given fractional number 1 $\bar{2}$ $\bar{4}$, taking $\bar{\bar{3}}$ and halving to get $\bar{3}$, and halving twice to get $\bar{4}$.

Problem 33

A quantity, its $\bar{\bar{3}}$, its $\bar{2}$, and its $\bar{7}$, added together, become 37. What is the quantity?

Multiply 1 $\bar{\bar{3}}$ $\bar{2}$ $\bar{7}$ so as to get 37.

1	1 $\bar{\bar{3}}$ $\bar{2}$ $\bar{7}$
2	4 $\bar{3}$ $\bar{4}$ $\bar{28}$
4	9 $\bar{6}$ $\bar{14}$
8	18 $\bar{3}$ $\bar{7}$
\ 16	36 $\bar{\bar{3}}$ $\bar{4}$ $\bar{28}$.

Applying $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{28}$ to 42 we have

1	42
\\$\frac{2}{3}$	28
$\frac{1}{2}$	21
\\$\frac{1}{4}$	10 $\frac{1}{2}$
\\$\frac{1}{28}$	1 $\frac{1}{2}$.[1]

The total is 40; there remains 2, or $\frac{1}{21}$ of 42. As 1 $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{7}$ applied to 42 gives 97, we shall have as a continuation of our first multiplication

$\frac{1}{97}$	$\frac{1}{42}$ or 1 as a part of 42
\\$\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$ $\frac{1}{21}$	" 2 " " " " 42.

This $\frac{1}{21}$ with the product already obtained will make the total 37. Thus the required quantity is 16 $\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$.

Proof.

1	16 $\frac{1}{56}$ $\frac{1}{679}$ $\frac{1}{776}$
$\frac{2}{3}$	10 $\frac{2}{3}$ $\frac{1}{84}$ $\frac{1}{1358}$ $\frac{1}{4074}$ $\frac{1}{1164}$
$\frac{1}{2}$	8 $\frac{1}{112}$ $\frac{1}{1358}$ $\frac{1}{1552}$
$\frac{1}{7}$	2 $\frac{1}{4}$ $\frac{1}{28}$ $\frac{1}{392}$ $\frac{1}{4753}$ $\frac{1}{5432}$.

The whole numbers and larger fractions make 36 $\frac{2}{3}$ $\frac{1}{4}$ $\frac{1}{28}$; the remainder is $\frac{1}{28}$ $\frac{1}{84}$. The smaller fractions applied to 5432 make

97	8	7	
64 $\frac{2}{3}$	4	1 $\frac{1}{3}$	4 $\frac{2}{3}$
48 $\frac{1}{2}$	4	3 $\frac{1}{2}$	
13 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{14}$ $\frac{1}{28}$	1 $\frac{1}{7}$	1.	

$\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{28}$, and the fractions of the remainder, $\frac{1}{28}$ and $\frac{1}{84}$, applied to 5432 make 3621 $\frac{1}{3}$, 1358, 194, and 194 and 64 $\frac{2}{3}$; for we have

1	5432
$\frac{2}{3}$	3621 $\frac{1}{3}$
$\frac{1}{2}$	2716
$\frac{1}{4}$	1358
$\frac{1}{28}$	194
Total	5173 $\frac{1}{3}$.

There remains 258 $\frac{2}{3}$ which is equal to 194 plus 64 $\frac{2}{3}$.

We may notice that in the course of his proof our author has $\frac{2}{3}$ of $\frac{1}{679}$ equal to $\frac{1}{1358}$ $\frac{1}{4074}$ a remarkable application of the rule given in Problem 61.

For an explanation of the proof see Introduction, pages 27–28. The calculation by which the 97 is obtained is given above in the solution of Problem 31.

[1] Since $\frac{2}{3}$ of 42 is 28, 1 $\frac{1}{2}$ being the reciprocal of $\frac{2}{3}$.

Problem 34

A quantity, its $\frac{1}{2}$, *and its* $\frac{1}{4}$, *added together, become* 10. *What is the quantity?*

Multiply 1 $\frac{1}{2}$ $\frac{1}{4}$ so as to get 10.

\\1	1 $\frac{1}{2}$ $\frac{1}{4}$
2	3 $\frac{1}{2}$
\\4	7
\\$\frac{1}{7}$	$\frac{1}{4}$ [1]
$\frac{1}{4}$ $\frac{1}{28}$	$\frac{1}{2}$
\\$\frac{1}{2}$ $\frac{1}{14}$	1.

The total is the quantity required, 5 $\frac{1}{2}$ $\frac{1}{7}$ $\frac{1}{14}$.

Proof.

\\1	5 $\frac{1}{2}$ $\frac{1}{7}$ $\frac{1}{14}$
\\$\frac{1}{2}$	2 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{14}$ $\frac{1}{28}$
\\$\frac{1}{4}$	1 $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{28}$ $\frac{1}{56}$.

The whole numbers and simpler fractions (powers of $\frac{1}{2}$) make a total of 9 $\frac{1}{2}$ $\frac{1}{8}$; the remainder is $\frac{1}{4}$ $\frac{1}{8}$. The remaining fractions, namely,

$\frac{1}{7}$ $\frac{1}{14}$ $\frac{1}{14}$ $\frac{1}{28}$ $\frac{1}{28}$ $\frac{1}{56}$,

applied to 56, are equal to

8 4 4 2 2 1,

making a total of 21, while $\frac{1}{4}$ and $\frac{1}{8}$ make 14 and 7, and so also a total of 21. Therefore the result obtained is correct.

The first step is a multiplication of the second kind (Introduction, page 5) with 1 $\frac{1}{2}$ $\frac{1}{4}$ taken as multiplicand and 10 as product. After doubling twice, also taking $\frac{1}{7}$ and doubling this twice, the author finds a combination of products exactly equal to 10, and it is not necessary for him to take the second and third steps that belong to this kind of multiplication when the product is not obtained directly from the partial products. It is easy to see why this multiplication should be so simple; for we have already enough partial products to make any integer up to 14. Compare the notes to Problem 37.

SECTION VII

Problems 35–38. Division of a *Hekat*

In these problems in the papyrus the questions are put in a curious way: "I have gone a certain number of times into the *hekat*-measure, certain parts have been added to me, and I return filled. What is it that says this?" It is stated as if the vessel represented as speaking had gone

[1] See Introduction, page 5, footnote 2.

into the *hekat*-measure and returned filled, but clearly it is the *hekat*-measure that is filled.

In Problem 36 the word *hekat* is omitted and the problem is stated and solved as a simple numerical problem. It is worded, however, in the same way as the other three except for the omission of the word *hekat* and it is solved in the same way. Also the proof is the same as the proof of the numerical result in each of the other three. In the three *hekat* problems the answer obtained as an ordinary fractional part of a *hekat* and proved as such is then reduced to *ro* and proved for the number of *ro*; and, finally, it is reduced to the "Horus eye" fractions as far as possible, and proved for this form of expression (see Introduction, page 31).

The method of solution of these problems is that of false position and the solution of Problem 35 is explained in full in the Introduction, page 11.

For an account of the *hekat* and its subdivisions see Introduction pages 31–33.

Problem 35

I have gone three times into the hekat-*measure, my* $\frac{1}{3}$ *has been added to me, and I return having filled the* hekat-*measure. What is it that says this?*

Do it thus: Assume 1. Multiplying by $3\ \dot{\frac{1}{3}}$ we have

\1	1
\2	2
\ $\frac{1}{3}$	$\frac{1}{3}$
Total	$3\ \frac{1}{3}$.

Get 1 by operating on $3\ \frac{1}{3}$.

1	$3\ \frac{1}{3}$
$\frac{1}{10}$	$\frac{1}{3}$
$\frac{1}{5}$	$\frac{2}{3}$
Total	1,

The answer is $\frac{1}{5}\ \frac{1}{10}$.

Proof.

\1	$\frac{1}{5}\ \frac{1}{10}$
\2	$\frac{1}{2}\ \frac{1}{10}$
\ $\frac{1}{3}$	$\frac{1}{10}$
Total	1.

Express the result in *ro*

1	320
$\frac{1}{10}$	32
$\frac{1}{5}$	64
Total	96.

Proof of the result as expressed in *ro*.

1	96
2	192
$\frac{1}{3}$	32
Total	320.

Expressed in "Horus eye" fractions[1] it makes of grain **1/4 1/32 1/64** *hekat* 1 *ro*.

Proof of the result expressed in this form.

1	**1/4 1/32 1/64** *hekat* 1 *ro*
2	**1/2 1/16 1/32** " 2 "
$\frac{1}{3}$	**1/16 1/32** " 2 "
Total	1.

Problem 36

I have gone in three times, my $\frac{1}{3}$ and my $\frac{1}{5}$ have been added to me, and I return having filled the measure. What is the quantity that says this?

Assume 1. Multiplying by 3 $\frac{1}{3}$ $\frac{1}{5}$ we have

1	1
1	1
1	1
$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{5}$	$\frac{1}{5}$
Total	3 $\frac{1}{3}$ $\frac{1}{5}$.

Get 1 by operating on 3 $\frac{1}{3}$ $\frac{1}{5}$. Apply this to 30; it makes 106. Multiply 106 so as to get 30.

1	106
$\frac{1}{2}$	53
\ $\frac{1}{4}$	26 $\frac{1}{2}$
\ $\frac{1}{106}$	1
\ $\frac{1}{53}$	2
\ $\frac{1}{212}$	$\frac{1}{2}$
Total	30,

[1] See Introduction, page 31. The "Horus eye" fractions are indicated by black-faced type throughout this work.

that is, the whole of 30, or 1. The answer is $\frac{1}{4}$ $\frac{1}{53}$ $\frac{1}{106}$ $\frac{1}{212}$.

Proof.

1	$\frac{1}{4}$ $\frac{1}{53}$ $\frac{1}{106}$ $\frac{1}{212}$
2	$\frac{1}{2}$ $\frac{1}{30}$ $\frac{1}{318}$ $\frac{1}{795}$ $\frac{1}{53}$ $\frac{1}{106}$
$\frac{1}{3}$	$\frac{1}{12}$ $\frac{1}{159}$ $\frac{1}{318}$ $\frac{1}{636}$
$\frac{1}{5}$	$\frac{1}{20}$ $\frac{1}{265}$ $\frac{1}{530}$ $\frac{1}{1060}$

The larger fractions are $\frac{1}{2}$ and $\frac{1}{4}$. In order to get 1 we should have for the sum of the remaining fractions $\frac{1}{4}$. To get this apply these fractions to 1060.

The fractions

	$\frac{1}{53}$	$\frac{1}{106}$	$\frac{1}{212}$				
as parts of 1060 make	20	10	5			or	35
	$\frac{1}{30}$	$\frac{1}{318}$	$\frac{1}{795}$	$\frac{1}{53}$	$\frac{1}{106}$		
make	35 $\frac{1}{3}$	3 $\frac{1}{3}$	1 $\frac{1}{3}$	20	10	"	70
	$\frac{1}{12}$	$\frac{1}{159}$	$\frac{1}{318}$	$\frac{1}{636}$			
"	88 $\frac{1}{3}$	6 $\frac{2}{3}$	3 $\frac{1}{3}$	1 $\frac{2}{3}$		"	100
	$\frac{1}{20}$	$\frac{1}{265}$	$\frac{1}{530}$	$\frac{1}{1060}$			
"	53	4	2	1		"	60

The total is 265, or $\frac{1}{4}$ of 1060; for

1	1060
$\frac{1}{2}$	530
$\frac{1}{4}$	265
$\frac{1}{4}$	265
Total	1060.

In multiplying 1 at the beginning of this solution by 3 $\frac{1}{3}$ $\frac{1}{5}$, instead of saying, once 1, twice 2, etc., our author actually writes down "once 1" three times and then the rest of the multiplication, and in getting 1 by operating on 3 $\frac{1}{3}$ $\frac{1}{5}$, instead of multiplying this expression directly by multipliers that will eventually give him 1, he applies it to 30, noting that 3 times 30 and $\frac{1}{3}$ $\frac{1}{5}$ of 30 make 106. Therefore to find how many times 3 $\frac{1}{3}$ $\frac{1}{5}$ will make 1 he determines how many times 106 will make 30 and the answer to this, $\frac{1}{4}$ $\frac{1}{53}$ $\frac{1}{106}$ $\frac{1}{212}$, is the answer to the problem.[1]

Problem 37

I have gone three times into the hekat*-measure, my $\frac{1}{3}$ has been added to me, $\frac{1}{3}$ of my $\frac{1}{3}$ has been added to me, and my $\frac{1}{9}$ has been added to me; I return having filled the* hekat*-measure. What is it that says this?*

[1] This is the only time that he applies his fractional expressions to a particular number for the purpose of dividing. Generally he uses the method for an addition or subtraction. See Introduction, page 9, footnote.

Assume 1. Multiplying by the given expression we have

1	1
2	2
$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$ of $\frac{1}{3}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$
Total	3 $\frac{1}{2}$ $\frac{1}{18}$.

Get 1 by operating on 3 $\frac{1}{2}$ $\frac{1}{18}$.

1	3 $\frac{1}{2}$ $\frac{1}{18}$
$\frac{1}{2}$	1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{36}$
\\ $\frac{1}{4}$	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{72}$
$\frac{1}{8}$	$\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{144}$
$\frac{1}{16}$	$\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{288}$
\\ $\frac{1}{32}$	$\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ $\frac{1}{576}$

The total, $\frac{1}{4}$ $\frac{1}{32}$, times, 3 $\frac{1}{2}$ $\frac{1}{18}$ makes 1, for we have first to add $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$, and then the smaller fractions

$\frac{1}{72}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ $\frac{1}{576}$,

which, taken as parts of 576, make

8 36 18 9 1,

a total of 72, or $\frac{1}{8}$ of 576. Therefore the answer is $\frac{1}{4}$ $\frac{1}{32}$.

Proof.

1	$\frac{1}{4}$ $\frac{1}{32}$
2	$\frac{1}{2}$ $\frac{1}{16}$
$\frac{1}{3}$	$\frac{1}{12}$ $\frac{1}{96}$
$\frac{1}{3}$ of $\frac{1}{3}$	$\frac{1}{36}$ $\frac{1}{288}$
$\frac{1}{9}$	$\frac{1}{36}$ $\frac{1}{288}$.

The total is 1, for we have first to add $\frac{1}{2}$ and $\frac{1}{4}$, and then the smaller fractions

$\frac{1}{32}$ $\frac{1}{16}$ $\frac{1}{12}$ $\frac{1}{96}$ $\frac{1}{36}$ $\frac{1}{288}$ $\frac{1}{36}$ $\frac{1}{288}$,

which, taken as parts of 288, make

9 18 24 3 8 1 8 1,

a total of 72, or $\frac{1}{4}$ of 288.

Express the result in *ro*.

1	320
$\frac{1}{2}$	160
\\ $\frac{1}{4}$	80
$\frac{1}{8}$	40
$\frac{1}{16}$	20
\\ $\frac{1}{32}$	10
Total	90.

Proof of the result as expressed in *ro*

\1	90
\2	180
\⅓	30
\⅓ of ⅓	10
\⅙	10
Total	320.

It amounts in grain to ¼ 1/32 *hekat*.

Proof of the result expressed in this form.

\1	¼ 1/32
\2	½ 1/16
\⅓	1/16 1/32
\⅓ of ⅓	1/32
\⅙	1/32
Total	½ ⅛ ¼ ⅛.

In this problem the *hekat* is to be divided by 3 ⅓ 1/18. The author expresses this number in the peculiar form 3 ⅓ ⅓of⅓ ⅙, but when he assumes 1 and multiplies 1 by this number he gets the simpler form, and this he uses in determining that it must be multiplied by ¼ 1/32 to get 1. In his proof, however, instead of taking 3 ⅓ 1/18, he very properly goes back to his original fractions as given in the statement of the problem, multiplying his answer by 3 ⅓ ⅓of⅓ ⅙.

The second operation is to multiply 3 ⅓ 1/18 so as to get 1. He proceeds by halving to obtain a series of fractional expressions, from which he finds at once a combination that makes exactly 1 (see Introduction, page 5). This would almost seem to indicate that the problem was made up from the answer. We find, however, that his procedure was most natural, and in no way does it indicate that he knew what would be the result. He wishes to get 1 or some number nearly equal to 1. The first two partial products are too large, but the third, ⅓ ¼ ⅙ 1/72 is less than 1 and can be used. Having selected this he might have proceeded at once to find the remainder by the process of completion, but he seems to have chosen to carry his multiplication further in the hope of getting still nearer 1. Since ⅓ ¼ ⅙, even without the 1/72, require only ¼ to make 1, any further expression that he could use must be less than ¼, and so he was compelled to pass over the next two partial products and the first one that he could try was the one beginning with 1/16. With this he finds that he gets exactly 1 and he does not have to proceed further. See notes to Problem 34.

After obtaining and proving the result expressed in the ordinary form as a fractional part of a *hekat* he reduces it to *ro* and then to the "Horus eye" fractions and proves for both of these forms that it is correct. As the denominators of the fractions that he first obtains are themselves powers of 2 he has the same fractions in his last expression, but he writes them in the "Horus eye" forms, and in his proof he has a slightly different combination because he takes ⅓ of ¼ 1/32 as 1/16 1/32 instead of 1/12 1/96, which the notation would not permit.

At the very end, instead of writing 1 *hekat* as the result of the multiplications of his proof, he writes its equivalent as ½ ⅛ ¼ ⅛.

Problem 38

I have gone three times into the hekat-*measure, my* $\frac{1}{7}$ *has been added to me, and I return having filled the* hekat-*measure. What is it that says this?*

Assume 1. Multiplying by the given expression we have

\1	1
\2	2
\ $\frac{1}{7}$	$\frac{1}{7}$
Total	3 $\frac{1}{7}$.

Get 1 by operating on 3 $\frac{1}{7}$.

1	3 $\frac{1}{7}$
$\frac{1}{22}$	$\frac{1}{7}$,

for $\frac{1}{7}$ of 22 is 3 $\frac{1}{7}$,

$\frac{1}{11}$	$\frac{1}{4}$ $\frac{1}{28}$
$\frac{1}{6}$ $\frac{1}{66}$	$\frac{1}{2}$ $\frac{1}{14}$
Total	1.

Therefore the answer is $\frac{1}{6}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$.

Proof.

1	$\frac{1}{6}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$
2	$\frac{1}{2}$ $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{66}$
$\frac{1}{7}$	$\frac{1}{22}$,

for $\frac{1}{22}$ of 7 is the expression that we have obtained.

Total	1.

Express the result in *ro.*

1	320
$\frac{2}{3}$	213 $\frac{1}{3}$
$\frac{1}{3}$	106 $\frac{2}{3}$
\ $\frac{1}{6}$	53 $\frac{1}{3}$
\ $\frac{1}{11}$	29 $\frac{1}{11}$
\ $\frac{1}{22}$	14 $\frac{1}{2}$ $\frac{1}{22}$
\ $\frac{1}{66}$	4 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{66}$
Total	101 $\frac{2}{3}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$.

Proof of the result as expressed in *ro.*

\1	101 $\frac{2}{3}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$
\2	203 $\frac{1}{2}$ $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{66}$
\ $\frac{1}{7}$	14 $\frac{1}{2}$ $\frac{1}{22}$
Total	320,

or 1 *hekat.*

It amounts in grain to $\frac{1}{4}$ $\frac{1}{16}$ *hekat* 1 $\frac{2}{3}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$ *ro.*

Proof of the result expressed in this form.

1	$\frac{1}{4}$ $\frac{1}{16}$	*hekat*	1 $\frac{2}{3}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$ *ro*
2	$\frac{1}{2}$ $\frac{1}{8}$	"	3 $\frac{1}{2}$ $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{66}$ "
$\frac{1}{7}$	$\frac{1}{32}$	"	4 $\frac{1}{2}$ $\frac{1}{22}$ "

The total of the larger portions of these products makes

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ *hekat* 4 $\frac{2}{3}$ *ro*,

or 319 $\frac{2}{3}$ *ro.*

The smaller fractions

$\frac{1}{11}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{22}$ $\frac{1}{33}$ $\frac{1}{66}$ $\frac{1}{66}$,

taken as parts of 66, are equal to

6 6 3 3 2 1 1.

The total is 22, which is $\frac{1}{3}$ of 66, and this with 319 $\frac{2}{3}$ makes 320 *ro* or 1 *hekat*, as it should.

There are some peculiarities here in the numerical work. Thus in multiplying 3 $\frac{1}{7}$ so as to get 1, the author starts by saying that $\frac{1}{22}$ of 3 $\frac{1}{7}$ is $\frac{1}{7}$, and his explanation, written on one side, is literally, "The making it is of $\frac{1}{7}$ times 22 to find 3 $\frac{1}{7}$." Also a little further along, in proving his answer, $\frac{1}{6}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$, multiplying it by 3 $\frac{1}{7}$, he writes as one step, $\frac{1}{7}$ $\frac{1}{22}$, and explains it by saying in almost the same words, "The making it is of $\frac{1}{22}$ times 7 to find the fraction above." It is as if he were familiar with the relation of the three numbers 7, 3 $\frac{1}{7}$ and 22. Possibly he had this relation in some table; or in some previous reckoning he had multiplied 7 "for the finding of 22," which would give him at once 3 $\frac{1}{7}$. Then he would know that $\frac{1}{7}$ of 22 is 3 $\frac{1}{7}$, that $\frac{1}{22}$ of 3 $\frac{1}{7}$ is $\frac{1}{7}$, and that $\frac{1}{22}$ of 7 is the same as the result of "getting 1 by operating on 3 $\frac{1}{7}$;" that is, it is the number that he gets in this solution, $\frac{1}{6}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$, so that, finally, $\frac{1}{7}$ of the last expression is equal to $\frac{1}{22}$.[1]

The fractional expressions in the solution and different proofs are quite complicated and it is possible that he applied them to 66 each time that he had additions to make, but, except in the last case, there is no indication of this.

In the first proof he doubles $\frac{1}{6}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$ and gets $\frac{1}{2}$ $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{66}$. Here twice $\frac{1}{6}$ is $\frac{1}{3}$, twice $\frac{1}{11}$ is $\frac{1}{6}$ $\frac{1}{66}$ (by the table at the beginning) and the $\frac{1}{3}$ and $\frac{1}{6}$ make $\frac{1}{2}$; finally, twice $\frac{1}{22}$ $\frac{1}{66}$ make $\frac{1}{11}$ $\frac{1}{33}$. In a very similar way in the second and third proofs he doubles the fractional expression $\frac{2}{3}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$.

In working out the expression of the result in *ro* he has $\frac{1}{11}$ of 320 equal to 29 $\frac{1}{11}$, and in the proof which follows, $\frac{1}{7}$ of 101 $\frac{2}{3}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$ is given as 14 $\frac{1}{2}$ $\frac{1}{22}$; also in the last proof $\frac{1}{7}$ of $\frac{1}{4}$ $\frac{1}{16}$ *hekat* 1 $\frac{2}{3}$ $\frac{1}{11}$ $\frac{1}{22}$ $\frac{1}{66}$ *ro* is given as $\frac{1}{32}$ *hekat* 4 $\frac{1}{2}$ $\frac{1}{22}$ *ro*. These are more difficult than most of the divisions in the papyrus, and the author does not state explicitly how he performed them.

[1] See Introduction, page 5, footnote 2.

SECTION VIII

PROBLEMS 39–40. DIVISION OF LOAVES. ARITHMETICAL PROGRESSION

Problem 39

Example of finding the difference of share when 100 *loaves are divided among* 10 *men,* 50 *in equal shares among* 6 *men and* 50 *in equal shares among* 4. *What is the difference of the shares?*

Multiply 4 so as to get 50.

1	4
\10	40
\2	8
\½	2

Total 12 ½.

Multiply 6 so as to get 50.

1	6
2	12
4	24
\8	48
\⅓	2

Total 8 ⅓.

Therefore each of the 4 will receive 12 ½ and each of the 6 will receive 8 ⅓, and the difference of share will be 4 ⅙.

In this problem the result obtained and the methods used are so simple that the problem seems hardly worth while. It is possible that the author intended to state a problem in arithmetical progression like the next one.

We may notice that he considers the 4 first in the solution, although he puts the 6 first in the statement. In the papyrus he expressed the result by writing 12 ½ four times and 8 ⅓ six times.

Problem 40

Divide 100 *loaves among* 5 *men in such a way that the shares received shall be in arithmetical progression and that* ⅐ *of the sum of the largest three shares shall be equal to the sum of the smallest two. What is the difference of the shares?*

Do it thus: Make the difference of the shares 5 ½. Then the amounts that the 5 men receive will be

23 17 ½ 12 6 ½ 1, total 60.

As many times as is necessary to multiply 60 to make 100, so many times must these terms be multiplied to make the true series.

\1	60
\⅔	40.

The total, 1 ⅔, times 60 makes 100.

Multiply by 1 ⅔

	23	it becomes	38 ⅓
	17 ½	" "	29 ⅙
	12	" "	20
	6 ½	" "	10 ⅔ ⅙
	1	" "	1 ⅔
Total	60	" "	100.

This problem has been fully explained in the Introduction (page 12) and needs no further comment here.

In this problem and in Problem 64 our author writes his progression as a descending series, and so the smallest term is with him the last term.

CHAPTER II. GEOMETRY

SECTION I

PROBLEMS 41–46. PROBLEMS OF VOLUME

Problem 41

Find the volume of a cylindrical granary of diameter 9 *and height* 10.

Take away $\frac{1}{9}$ of 9, namely, 1; the remainder is 8. Multiply 8 times 8; it makes 64. Multiply 64 times 10; it makes 640 cubed cubits. Add $\frac{1}{2}$ of it to it; it makes 960, its contents in *khar*. Take $\frac{1}{20}$ of 960, namely 48. 4800 *hekat* of grain will go into it.

Method of working out:

1	8
2	16
4	32
\8	64.

1	64
\10	640
\$\frac{1}{2}$	320
Total	960
$\frac{1}{10}$	96
\$\frac{1}{20}$	48.

As explained in the Introduction (pages 35–36), the author, in order to obtain the volume of the cylinder, subtracts from the diameter its $\frac{1}{9}$, squares the remainder, and multiplies by the altitude. In this way he finds that 640 is the contents of the granary in cubed cubits. As a *khar* is $\frac{2}{3}$ of a cubed cubit he adds to 640 its $\frac{1}{2}$, which gives him 960 as the number of *khar* in the granary. Finally, dividing by 20, he obtains 48 as the number of hundreds of quadruple *hekat*. (See Introduction, page 32.)

Problem 42

Find the volume of a cylindrical granary of diameter 10 *and height* 10.

Take away $\frac{1}{9}$ of 10, namely 1 $\frac{1}{9}$; the remainder is 8 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{18}$. Multiply 8 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{18}$ times 8 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{18}$; it makes 79 $\frac{1}{108}$ $\frac{1}{324}$. Multiply 79 $\frac{1}{108}$ $\frac{1}{324}$

times 10; it makes 790 $\frac{1}{18}$ $\frac{1}{27}$ $\frac{1}{54}$ $\frac{1}{81}$ cubed cubits. Add $\frac{1}{2}$ of it to it; it makes 1185 $\frac{1}{6}$ $\frac{1}{54}$, its contents in *khar*. $\frac{1}{20}$ of this is 59 $\frac{1}{4}$ $\frac{1}{108}$. 59 $\frac{1}{4}$ $\frac{1}{108}$ times 100 *hekat* of grain will go into it.

Method of working out:

1	8 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{18}$
2	17 $\frac{2}{3}$ $\frac{1}{9}$
4	35 $\frac{1}{2}$ $\frac{1}{18}$
\8	71 $\frac{1}{9}$
\\$\frac{2}{3}$	5 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{18}$ $\frac{1}{27}$
$\frac{1}{3}$	2 $\frac{2}{3}$ $\frac{1}{6}$ $\frac{1}{12}$ $\frac{1}{36}$ $\frac{1}{54}$
\\$\frac{1}{6}$	1 $\frac{1}{3}$ $\frac{1}{12}$ $\frac{1}{24}$ $\frac{1}{72}$ $\frac{1}{108}$
\\$\frac{1}{18}$	$\frac{1}{3}$ $\frac{1}{9}$ $\frac{1}{27}$ $\frac{1}{108}$ $\frac{1}{324}$
Total	79 $\frac{1}{108}$ $\frac{1}{324}$.

1	79 $\frac{1}{108}$ $\frac{1}{324}$
10	790 $\frac{1}{18}$ $\frac{1}{27}$ $\frac{1}{54}$ $\frac{1}{81}$
$\frac{1}{2}$	395 $\frac{1}{36}$ $\frac{1}{54}$ $\frac{1}{108}$ $\frac{1}{162}$
Total	1185 $\frac{1}{6}$ $\frac{1}{54}$
$\frac{1}{10}$	118 $\frac{1}{2}$ $\frac{1}{54}$
\\$\frac{1}{20}$	59 $\frac{1}{4}$ $\frac{1}{108}$.

This problem is exactly like 41, but with 10 instead of 9 for diameter there are many fractions. The $\frac{1}{108}$ of 100 *hekat* should be reduced to "Horus eye" fractions and *ro*, but in the papyrus this fraction is omitted.

Problem 43

A cylindrical granary of diameter 9 *and height* 6. *What is the amount of grain that goes into it?*

Do it thus: Take away $\frac{1}{9}$ of it, namely, 1, from 9; the remainder is 8. Add to 8 its $\frac{1}{3}$; it makes 10 $\frac{2}{3}$. Multiply 10 $\frac{2}{3}$ times 10 $\frac{2}{3}$; it makes 113 $\frac{2}{3}$ $\frac{1}{9}$. Multiply 113 $\frac{2}{3}$ $\frac{1}{9}$ times 4, 4 cubits being $\frac{2}{3}$ of the height; it makes 455 $\frac{1}{9}$, its contents in *khar*. Find $\frac{1}{20}$ of this, namely, 22 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{180}$. The amount of grain that will go into it is 22 $\frac{1}{2}$ $\frac{1}{4}$ times 100 *hekat* $\frac{1}{2}$ $\frac{1}{32}$ $\frac{1}{64}$ *hekat* 2 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{36}$ *ro*.

Method of working out:

\1	8
$\frac{2}{3}$	5 $\frac{1}{3}$
\\$\frac{1}{3}$	2 $\frac{2}{3}$
Total	10 $\frac{2}{3}$.

1	$10\ \frac{2}{3}$
\10	$106\ \frac{2}{3}$
\\$\frac{2}{3}$	$7\ \frac{1}{9}$
Total	$113\ \frac{2}{3}\ \frac{1}{9}$.

1	$113\ \frac{2}{3}\ \frac{1}{9}$
2	$227\ \frac{1}{2}\ \frac{1}{18}$
\4	$455\ \frac{1}{9}$.

1	$455\ \frac{1}{9}$
$\frac{1}{10}$	$45\ \frac{1}{2}\ \frac{1}{90}$
\\$\frac{1}{20}$	$22\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{180}$.

This solution for a long time baffled the ingenuity of Egyptologists, but the correct interpretation was finally discovered by Schack-Schackenburg (1899, see Peet, page 83).

In the first place, the papyrus states that the height of the granary is 9 and the breadth (diameter) 6, and in the solution, when we find 4 as $\frac{2}{3}$ of 6, the author again calls 6 the breadth; but the solution is for a cylinder in which 9 is the diameter and 6 the height.[1]

Then the method of solution is not that used in 41 and 42, but a second one (employed for a similar problem in the Kahun papyrus, Griffith, 1897), giving the volume directly in *khar* and not first in cubed cubits. It may be expressed in the following rule: Add to the diameter its $\frac{1}{3}$; square, and multiply by $\frac{2}{3}$ of the height. In Problem 43 the addition of its $\frac{1}{3}$ to the diameter makes 12, the square is 144, and $\frac{2}{3}$ of the height is 4. With these numbers the rule gives 576 *khar*, and this is just what the author would have obtained if he had followed the solutions of 41 and 42.[2]

But the author, before taking the steps of this rule, deducts from the diameter its $\frac{1}{9}$, as by the other rule, and so obtains a result which is $(\frac{8}{9})^2$ of the correct result, namely, $455\ \frac{1}{9}$ *khar*.[3]

[1] There is some confusion in Peet's explanation of this mistake (page 84). He supposes that the statement was correct in the original papyrus, but that "a later scribe, seeing in the first line of working the subtraction of a ninth of 9 from 9, . . . concluded that 9 must be the diameter and not the height, and so he transposed the two dimensions in the statement," but if the statement now in the papyrus is the result of such a transposition the transposition must have been the other way.

[2] In modern form the number of *khar* according to the two methods is expressed by the formulae $(\frac{8}{9}d)^2\ h.\frac{3}{2}$ and $(\frac{4}{3}d)^2.\frac{2}{3}\ h$, both of which reduce to $\frac{32}{27}\ hd^2$.

[3] Eisenlohr translated the word now read *khar* as "bodily content" and supposed that the addition of $\frac{1}{2}$ to reduce cubed cubits to *khar* was a part of the calculation of the cubed cubits. This is the same as multiplying the base by $\frac{3}{2}$ of the height instead of by the height itself, and to explain this he supposes that the given base in 41 and 42 is the upper base, and that the lower base is larger. When he comes to Problem 43 he has great difficulty. In the first place, he finds that $\frac{8}{9}$ of the diameter of the base is multiplied by $\frac{4}{3}$ ($\frac{1}{3}$ of it added) before squaring, as if the base were an ellipse with one axis $\frac{16}{9}$ of the other, or as if our author intended to get the lower base or some section between the two. Then the area obtained is multiplied by $\frac{2}{3}$ of the height instead of $\frac{3}{2}$, which might be because he had taken

In the last part of the solution the author expresses $\frac{1}{80}$ of 100 quadruple *hekat* in fractional parts of a quadruple *hekat* ("Horus eye" fractions) and quadruple *ro* and fractional parts. 100 times $\frac{1}{80}$ gives him $\frac{1}{2}$ $\frac{1}{32}$ $\frac{1}{64}$ and a remainder that must be multiplied further by 320 to reduce it to quadruple *ro*.

Problem 44

Example of reckoning the volume of a rectangular granary, its length being 10, *its width* 10, *and its height* 10. *What is the amount of grain that goes into it?*

Multiply 10 times 10; it makes 100. Multiply 100 times 10; it makes 1000. Add its $\frac{1}{2}$; it makes 1500, its contents in *khar*. Take $\frac{1}{20}$ of 1500; it makes 75, its contents in quadruple *hekat*, namely, 7500 *hekat* of grain.

The working out:

1	10
10	100
1	100
10	1000
1	1000
$\frac{1}{2}$	500
1	1500
$\frac{1}{10}$	150
$\frac{1}{20}$	75.

Proof:

1	75
10	750
\20	1500
$\frac{1}{10}$ of 1500	150
$\frac{1}{10}$ of $\frac{1}{10}$	15
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$	10.

This problem is similar to 41 and 42 except that the base is a square instead of a circle. At the end, after the author has completed his calculations, he goes through them in reverse order, apparently as a proof of the correctness of his result. This reverse calculation is the same as the calculation given as the solution of the next problem, the next problem being the reverse of the present one.

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the larger base, or because he had some vague idea of the rule for the volume of a hemisphere (the area of the base times $\frac{2}{3}$ of the height), and applied it to this solid, even though it may not have been spherical. This is the way in which the question of a hemisphere came into the problem. See titles of articles by Borchardt (1897) and Schack-Schackenburg (1899).

Problem 45

A rectangular granary into which there have gone 7500 *quadruple* hekat *of grain. What are its dimensions?*

Multiply 75 times 20; it makes 1500. Take $\frac{1}{10}$ of 1500, namely, 150, $\frac{1}{10}$ of its $\frac{1}{10}$, 15, $\frac{2}{3}$ of $\frac{1}{10}$ of its $\frac{1}{10}$, 10. Therefore the dimensions are 10 by 10 by 10.

The working out:

1	75
10	750
20	1500,

its contents in *khar.*

1	1500
$\frac{1}{10}$	150
$\frac{1}{10}$ of $\frac{1}{10}$	15
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$	10.

Instead of taking $\frac{2}{3}$ at the beginning to reduce the contents to cubed cubits, as he would have done if he had exactly reversed the process of the preceding solution, the author takes $\frac{2}{3}$ of the last quotient to find the third dimension.

Problem 46

A rectangular granary into which there have gone 2500 *quadruple* hekat *of grain. What are its dimensions?*

Multiply 25 times 20; it makes 500, its contents in *khar*. Take $\frac{1}{10}$ of 500, namely, 50, its $\frac{1}{20}$, 25, $\frac{1}{10}$ of its $\frac{1}{10}$, 5, $\frac{2}{3}$ of $\frac{1}{10}$ of its $\frac{1}{10}$, 3 $\frac{1}{3}$. Therefore the dimensions are 10 by 10 by 3 $\frac{1}{3}$.

Its working out:

1	25
10	250
20	500,

its contents in *khar.*

1	500
$\frac{1}{10}$	50
$\frac{1}{10}$ of $\frac{1}{10}$	5
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$	3 $\frac{1}{3}$.

The dimensions of the granary are therefore in cubits 10 by 10 by 3$\frac{1}{3}$.

The $\frac{1}{20}$ that the author takes in this solution is not necessary and is not used.

SECTION II

Problem 47. Division of 100 *Hekat*

Problem 47

Suppose the scribe says to thee, Let me know what is the result when 100 *quadruple* hekat *are divided by* 10 *and its multiples, in a rectangular or circular granary.*

$\frac{1}{10}$	becomes	of grain	10 quadruple *hekat*					
$\frac{1}{20}$	"	"	"	5	*hekat*			
$\frac{1}{30}$	"	"	"	3 $\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{64}$	"	1 $\frac{2}{3}$	*ro*
$\frac{1}{40}$	"	"	"	2 $\frac{1}{2}$	"		
$\frac{1}{50}$	"	"	"	2	"		
$\frac{1}{60}$	"	"	"	1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{32}$	"	3 $\frac{1}{3}$	"
$\frac{1}{70}$	"	"	"	1 $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{32}$ $\frac{1}{64}$	"	2 $\frac{1}{14}$ $\frac{1}{21}$ $\frac{1}{42}$	"
$\frac{1}{80}$	"	"	"	1 $\frac{1}{4}$	"		
$\frac{1}{90}$	"	"	"	1 $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$	"	$\frac{1}{2}$ $\frac{1}{18}$	"
$\frac{1}{100}$	"	"	"	1	"		

In the original the scribe is made to ask only for $\frac{1}{10}$, but the author gives $\frac{1}{10}$, $\frac{1}{20}$, etc.

SECTION III

Problems 48–55. Problems of Area

Problem 48

Compare the area of a circle and of its circumscribing square.

The circle of diameter 9.		The square of side 9.	
1	8 *setat*	\1	9 *setat*
2	16 "	2	18 "
4	32 "	4	36 "
\8	64 "	\8	72 "
		Total	81 "

Problem 49

Example of reckoning area. Suppose it is said to thee, What is the area of a rectangle of land of 10 khet *by* 1 khet?

Do it thus:

1	1,000
10	10,000
100	100,000
$\frac{1}{10}$	10,000
$\frac{1}{10}$ of $\frac{1}{10}$	1,000.

This is its area.

The papyrus states the problem for a field of 10 *khet* by 2 *khet*, and these numbers are in the figure, but the solution is for 10 *khet* by 1 *khet*, or 1,000 cubits by 100 cubits. Multiplying these numbers together gives 100,000 square cubits. Dividing this by 100 gives 1,000 cubit-strips, strips 1 cubit wide and 1 *khet* long.

Problem 50

Example of a round field of diameter 9 khet. *What is its area?*

Take away $\frac{1}{9}$ of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 *setat* of land.

Do it thus:

	1	9
	$\frac{1}{9}$	1;
this taken away leaves 8		
	1	8
	2	16
	4	32
	\8	64.

Its area is 64 *setat.*

Problem 51

Example of a triangle of land. Suppose it is said to thee, What is the area of a triangle of side[1] 10 khet *and of base* 4 khet?

Do it thus:

1	400
$\frac{1}{2}$	200
1	1,000
2	2,000.

Its area is 20 *setat.*

Take $\frac{1}{2}$ of 4, in order to get its rectangle. Multiply 10 times 2; this is its area.

The author seems to put the reckoning before the explanation. In the reckoning he puts down the base as 400 and the side as 1,000; that is, he expresses these lengths in cubits. Dividing 400 by 2 he gets 200 and 1,000 as the dimensions of the equivalent rectangle. Then to obtain the area expressed as so many cubit-strips he multiplies 1,000, not by 200, but by 2, as if he thought of the rectangle as made up of 1,000 pairs of cubit-strips. Finally, he writes down 2, that is, 20 *setat* (2 ten-*setat*), as the standard form of expressing the result.

[1] For a discussion of the question whether this word should be *side* or *altitude* see Introduction, pages 36–37, and Bibliography, pages 132–134.

In his explanation he uses the number of *khet* in the base and the number of *khet* in the side, and says that 10 times 2 will give the area.

A similar problem is given on a fragment of the Golenishchev papyrus (see Bibliography under Tsinserling, 1925).

Problem 52

Example of a cut-off (truncated) triangle of land. Suppose it is said to thee, What is the area of a cut-off triangle of land of 20 khet *in its side,* 6 khet *in its base,* 4 khet *in its cut-off line?*

Add its base to its cut-off line; it makes 10. Take ½ of 10, that is 5, in order to get its rectangle. Multiply 20 times 5; it makes 10 (10 ten-*setat*). This is its area.

Do it thus:

1	1,000
½	500
\1	2,000
2	4,000
\4	8,000
Total	10,000.

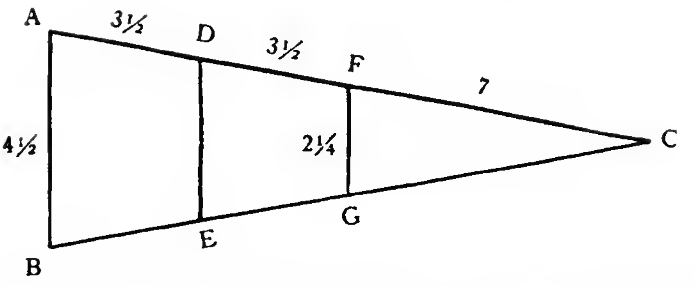
Its area is 100 *setat* (10 ten-*setat*).

This solution is very much like the preceding. Taking ½ of the sum of the bases he gets 500 cubits or 5 *khet* as the base of the equivalent rectangle with its side 2,000 cubits. Multiplying 2,000 by 5 he gets 10,000 cubit-strips for the area. Finally he expresses this in *setat* (or ten-*setat*). In the papyrus 20 was written where 10 should have been.

Problem 53

Areas of sections of a triangle.

\1	4 ½	*setat*	
\2	9	"	
\½	2 ¼	"	
Total	15 ½ ¼	"	
1/10	1 ½	"	7 ½ cubit-strips.
Its 1/10 taken away leaves the area	14 ⅛	"	5 "
1	7	"	
\2	14	"	
½	3 ½	"	
\¼	1 ½ ¼	"	
Total	15 ½ ¼	"	
½	7 ½ ¼ ⅛	"	



This problem is difficult to explain, and the difficulty is increased by some numerical mistakes. The most probable explanation that I can give is that the author is undertaking to determine the areas of certain sections of an isosceles triangle. The drawing suggests that 14 is the common length of the two equal sides and that the sections are made by two lines parallel to the base. One of these lines seems to bisect the two equal sides and perhaps it was intended that the other should bisect the parts next to the base, so that the lengths of the sides of the three sections would be 3 ½, 3 ½, and 7. The length of the shorter dividing line is put down as 2 ¼, and this would make the base 4 ½ and the longer dividing line 3 ¼ ⅛. In his figure the author puts down 6 as the length of both of these lines, but in his calculations he seems to take 4 ½ for the base.

He first undertakes to determine the area of the largest section. Apparently he intended to multiply the base, 4 ½, by the height or side 3 ½. This would give him 15 ½ ¼ *setat*. He recognizes that the area thus determined would be too large, because the top is less than the base. He does not seem able to find the length of the upper line, but proposes arbitrarily to take away 1/10 of the area that he has obtained. In this way, if he had finished his solution, he would have had, finally, as the area, 14 ⅛ *setat* 5 cubit-strips.[1] This taking away of 1/10 reminds us of his solution of Problem 28 and of his method of determining the area of a circle by taking away 1/9 of the diameter. In Problem 82 we shall have an example where he obtains a quantity somewhat smaller than a given quantity by taking away 1/10 of ⅔ of it.

The true area, if 3 ½ were the height, would be easily found. AB and FG being 4 ½ and 2 ¼,

AB + FG will be	6 ½ ¼
Its ½, DE	3 ¼ ⅛
Adding AB we get	7 ½ ¼ ⅛
Its ½	3 ½ ¼ ⅛ 1/16,

one-half of the sum of the bases of the trapezoid ABED. Multiplying by 3½ we have

\1	3 ½ ¼ ⅛ 1/16
\2	7 ½ ¼ ⅛
\½	1 ½ ¼ ⅛ 1/16 1/32
Total	13 ½ ¼ 1/32.

[1] Actually the author multiplied 4 ½ by 1, 2, ½ and ¼, checking the 1 and 2. Then he put down as his total 5 ½ ⅛, which is the sum of the first and last of these partial products. Finally he put down as 1/10, 1 ¼ ⅛ *setat* 10 cubit-strips. This is very nearly 1/10 of 15 ½ ¼, and much too large to be 1/10 of his 5 ½ ⅛.

This result expressed in standard form (see Introduction, page 33) is 13½ ¼ *setat* 3⅛ cubit-strips. The result obtained by the author's method exceeds this by 63/100 *setat*, or a little less than 3%.

He next determines the area of the apex. To do this he multiplies 7 by 2¼ and divides by 2, getting 7½ ¼ ⅛ *setat* as this area.

Finally, to determine the area of the middle section he would simply have to subtract the areas of the other two sections from the area of the whole triangle.

Problems 54 and 55 are exactly alike, although the first is given in the papyrus more briefly and with some words missing. I will explain them together.

Problem 54

What equal areas should be taken from 10 *fields if the sum of these areas is to be* 7 setat?

Multiply 10 so as to get 7.

1	10
\\½	5
\\⅕	2

Total ½ ⅕.[1]

Expressed as parts of a *setat* and cubit-strips this is ½ ⅛ *setat* 7 ½ cubit-strips.

Proof.

1	½ ⅛	*setat*	7 ½	cubit-strips
\\2	1 ¼ ⅛	"	2 ½	"
4	2 ½ ¼	"	5	"
\\8	5 ½	"	10	"
Total	7	"		

Problem 55

What equal areas should be taken from 5 *fields if the sum of these areas is to be* 3 setat?

Multiply 5 so as to get 3.

1	5
½	2 ½
1/10	½

Total ½ 1/10.

Expressed as parts of a *setat* and cubit-strips this is ½ *setat* 10 cubit-strips.

[1] In Problem 4 this is given as ⅔ 1/30. See notes to that problem.

Proof.

\1	1/2	*setat*	10	cubit-strips
2	1 1/8	"	7 1/2	"
\4	2 1/4 1/8	"	2 1/2	"
Total	3.			

Thus you find that the area is 3 *setat.*

These two problems are simple division problems—10 is multiplied so as to get 7, and 5 so as to get 3—and they have been translated both by Eisenlohr and by Peet, "Divide . . . into . . . fields." But the preposition sometimes means *from* and does not mean *into*, and the verb at the beginning, which is used several times in the papyrus, elsewhere always means *take away* or *subtract.* Gunn (page 133) has suggested that these words can be used here with their ordinary meanings in the sense of taking away an equal part from each field.

In each of these problems a product and multiplier are given to find the multiplicand. Problem 54 is, How large a field taken 10 times (once from each of the given 10 fields) will make 7 *setat,* and Problem 55, How large a field taken 5 times will make 3 *setat?* As the Egyptian cannot solve these problems directly, he forms new ones in which the multipliers in these become multiplicands and the answers are obtained first as multipliers (see Introduction, page 6). In writing down the multiplications of these new problems he writes all of his numbers as mere numbers, but in Problem 55 he writes first the statement of his new problem as a problem in *setat.*[1] The answer to this new problem is 1/2 1/10, and if it is taken as a problem in *setat,* the argument for the answer to the given problem will be, 1/2 1/10 times 5 *setat* makes 3 *setat,* therefore 5 times 1/2 1/10 of a *setat* (or, as he has to write it, 1/2 *setat* 10 cubit-strips) will be 3 *setat,* and so the answer to the given problem is 1/2 *setat* 10 cubit-strips.

SECTION IV

Problems 56–60. Pyramids; the Relation of the Lengths of Two Sides of a Triangle

Problem 56

If a pyramid is 250 cubits high[2] *and the side of its base 360 cubits long, what is its* seked?

Take 1/2 of 360; it makes 180. Multiply 250 so as to get 180; it makes 1/2 1/5 1/50 of a cubit. A cubit is 7 palms. Multiply 7 by 1/2 1/5 1/50.

[1] He says, "Multiply 5 *setat* so as to get fields of 3 *setat.*" See Literal Translation. Peet (page 96) says that the Egyptian "illogically" divides 3 *setat* by 5 *setat.* But it would seem to be as logical to make the new problem one of *setat* as to make it one of mere numbers. Peet explains that it is impossible for the Egyptian to obtain the quotient otherwise than as a mere number; that the Egyptian can only divide 3 by 5 (so he says), giving 1/2 1/10; and then he adds, "Thus in the present case an Egyptian could not mark 1/2 1/10 as *setat,* because the *setat*-notation does not recognize such a quantity." But the important question here is, How can 1/2 1/10 be regarded as 1/2 1/10 of a *setat?*

[2] For a discussion of the terms used in these problems see Introduction, pages 37–38.

1	7
$\frac{1}{2}$	3 $\frac{1}{2}$
$\frac{1}{5}$	1 $\frac{1}{3}$ $\frac{1}{15}$
$\frac{1}{50}$	$\frac{1}{10}$ $\frac{1}{25}$

The *seked* is 5 $\frac{1}{25}$ palms.

Problem 57

If the seked *of a pyramid is* 5 *palms* 1 *finger per cubit and the side of its base* 140 *cubits, what is its altitude?*

Divide 1 cubit by the *seked* doubled, which is 10 $\frac{1}{2}$. Multiply 10 $\frac{1}{2}$ so as to get 7, for this is a cubit: 7 is $\frac{2}{3}$ of 10 $\frac{1}{2}$. Operate on 140, which is the side of the base: $\frac{2}{3}$ of 140 is 93 $\frac{1}{3}$. This is the altitude.

In this inverse problem and in 59B the author doubles the *seked* instead of taking $\frac{1}{2}$ of the side of the base, and instead of dividing the *seked* doubled by 7 and dividing the side of the base by the result, he divides 7 by the *seked* doubled and multiplies the side of the base by the result, which amounts to the same thing.

Problem 58

If a pyramid is 93 $\frac{1}{3}$ *cubits high and the side of its base* 140 *cubits long, what is its* seked?

Take $\frac{1}{2}$ of 140, which is 70. Multiply 93 $\frac{1}{3}$ so as to get 70. $\frac{1}{2}$ is 46 $\frac{2}{3}$, $\frac{1}{4}$ is 23 $\frac{1}{3}$. Make thou $\frac{1}{2}$ $\frac{1}{4}$ of a cubit. Multiply 7 by $\frac{1}{2}$ $\frac{1}{4}$. $\frac{1}{2}$ of 7 is 3 $\frac{1}{2}$, $\frac{1}{4}$ is 1 $\frac{1}{2}$ $\frac{1}{4}$, together 5 palms 1 finger. This is its *seked.*

The working out:

1	93 $\frac{1}{3}$
\\ $\frac{1}{2}$	46 $\frac{2}{3}$
\\ $\frac{1}{4}$	23 $\frac{1}{3}$
Total $\frac{1}{2}$ $\frac{1}{4}$.	

Make thou $\frac{1}{2}$ $\frac{1}{4}$ of a cubit; a cubit is 7 palms.

1	7
$\frac{1}{2}$	3 $\frac{1}{2}$
$\frac{1}{4}$	1 $\frac{1}{2}$ $\frac{1}{4}$
Total	5 palms 1 finger.

This is its *seked.*

Problem 59

If a pyramid is 8 *cubits high and the side of its base* 12 *cubits long, what is its* seked?

Multiply 8 so as to get 6, for this is $\frac{1}{2}$ of the side of the base.

1	8
\ $\frac{1}{2}$	4
\ $\frac{1}{4}$	2

Total $\frac{1}{2}$ $\frac{1}{4}$.

Take $\frac{1}{2}$ $\frac{1}{4}$ of 7; this is a cubit.

1	7
\ $\frac{1}{2}$	3 $\frac{1}{2}$
\ $\frac{1}{4}$	1 $\frac{1}{2}$ $\frac{1}{4}$.

The result is 5 palms 1 finger. This is its *seked*.

As in Problem 43, the two numbers given are interchanged in the statement in the papyrus, or we might say that the side of the base and the altitude are interchanged; and then once in the solution "altitude" is written for "side of the base."

The original contains two or three words at the end of the solution that are uncertain. See Literal Translation.

The next problem is the inverse of the preceding and Eisenlohr counted the two together as 59. Indeed, the author may have regarded them as one problem, since none of the second is written in red. In order to hold to the numbering of Eisenlohr I have followed Peet in calling it 59B.

Problem 59B

If the seked *of a pyramid is* 5 *palms* 1 *finger per cubit and the side of its base* 12 *cubits long, what is its altitude?*

Multiply 5 palms 1 finger doubled, which is 10 $\frac{1}{2}$, so as to get 1 cubit; a cubit is 7 palms. $\frac{2}{3}$ of 10 $\frac{1}{2}$ is 7; therefore $\frac{2}{3}$ of 12, which is 8, is the altitude.

Problem 60

If a pillar (?) [1] *is* 30 *cubits high and the side (diameter?) of its base* 15 *cubits, what is its* seked?

Take $\frac{1}{2}$ of 15; it is 7 $\frac{1}{2}$. Multiply 30 so as to get 7 $\frac{1}{2}$; the result is $\frac{1}{4}$. This is the *seked*.

The working out:

1	15
\ $\frac{1}{2}$	7 $\frac{1}{2}$

1	30
$\frac{1}{2}$	15
\ $\frac{1}{4}$	7 $\frac{1}{2}$.

[1] For a discussion of the meaning of this word see Peet, page 101.

In the papyrus, instead of dividing the base of the right triangle by the other given line, the author divides the other given line by the base. I follow Borchardt (1893) in treating this as a mistake.

At the end he does not multiply by 7 so as to express the *seked* in palms, as he does in the other problems.

CHAPTER III. MISCELLANEOUS PROBLEMS

Problem 61

Table for multiplication of fractions.

$\frac{2}{3}$	of	$\frac{2}{3}$	is	$\frac{1}{3}$ $\frac{1}{9}$
$\frac{1}{3}$	"	$\frac{2}{3}$	"	$\frac{1}{6}$ $\frac{1}{18}$
$\frac{2}{3}$	"	$\frac{1}{3}$	"	$\frac{1}{6}$ $\frac{1}{18}$
$\frac{2}{3}$	"	$\frac{1}{6}$	"	$\frac{1}{12}$ $\frac{1}{36}$
$\frac{2}{3}$	"	$\frac{1}{2}$	"	$\frac{1}{3}$
$\frac{1}{3}$	"	$\frac{1}{2}$	"	$\frac{1}{6}$
$\frac{1}{6}$	"	$\frac{1}{2}$	"	$\frac{1}{12}$
$\frac{1}{12}$	"	$\frac{1}{2}$	"	$\frac{1}{24}$

$\frac{1}{9}$ of $\frac{2}{3}$ is $\frac{1}{18}$ $\frac{1}{54}$; $\frac{1}{9}$, $\frac{2}{3}$ of it is $\frac{1}{18}$ $\frac{1}{54}$

. . . .

$\frac{1}{5}$, $\frac{1}{4}$ of it is $\frac{1}{20}$

$\frac{1}{7}$, $\frac{2}{3}$ " " " $\frac{1}{14}$ $\frac{1}{42}$

$\frac{1}{7}$, $\frac{1}{2}$ " " " $\frac{1}{14}$

$\frac{1}{11}$, $\frac{2}{3}$ " " " $\frac{1}{22}$ $\frac{1}{66}$, $\frac{1}{3}$ of it is $\frac{1}{33}$

$\frac{1}{11}$, $\frac{1}{2}$ " " " $\frac{1}{22}$, $\frac{1}{4}$ " " " $\frac{1}{44}$.

Peet points out (page 103) that the above table contains two forms of statement which have an interesting significance. In the first four lines we find forms of the type, $\frac{2}{3}$ of $\frac{2}{3}$ is $\frac{1}{3}$ $\frac{1}{9}$, while in the last five are such forms as, $\frac{1}{5}$, $\frac{1}{4}$ of it is $\frac{1}{20}$. In the ninth line both forms are given, the statement being made twice, while the four lines that immediately precede seem to have been originally in the second form but to have been changed to the first; see Literal Translation. The reason for the second form is that of the fractions in this table the only ones that were legitimate multipliers are $\frac{2}{3}$ and $\frac{1}{3}$ and fractions obtained from them by halving. Thus the Egyptian could not directly say, "$\frac{1}{5}$ of $\frac{1}{4}$" but only "$\frac{1}{5}$, $\frac{1}{4}$ of it." He could have used the second form in all of these cases, but when he has a legitimate multiplier he prefers the first form, and so the author, perceiving after they were written that lines 5–8, like the first four, involve only these multipliers, changed them to the first form.

Problem[1] *61B*

Rule for getting $\frac{2}{3}$ of the reciprocal of an odd number.

To get $\frac{2}{3}$ of $\frac{1}{5}$ take the reciprocals of 2 times 5 and 6 times 5, and in the same way get $\frac{2}{3}$ of the reciprocal of any odd number.

[1] See Introduction, pages 24–25.

Problem 62

Example of reckoning the contents of a bag of various precious metals. Suppose it is said to thee, A bag containing equal weights of gold, silver and lead, has been bought for 84 sha'ty. *What is the amount in it of each precious metal, that which is given for a* deben *of gold being* 12 sha'ty, *for a* deben *of silver* 6 sha'ty, *and for a* deben *of lead* 3 sha'ty?

Add that which is given for a *deben* of each precious metal. The result is 21 *sha'ty*. Multiply 21 so as to get 84, the 84 *sha'ty* for which this bag was bought. The result is 4, which is the number of *deben* of each precious metal.

Do it thus:

Multiply 12 by 4 getting 48 *sha'ty* for the gold in the bag,
Multiply 6 by 4 getting 24 *sha'ty* for the silver,
Multiply 3 by 4 getting 12 *sha'ty* for the lead,
Multiply 21 by 4 getting 84 *sha'ty* altogether.

The *sha'ty* was a seal, and the word here represents a unit of value (see Weill, 1925). The *deben* was a unit of weight, equal to about 91 grammes. The papyrus does not say that the bag contains equal weights of gold, silver, and lead, but in the solution the author proceeds as if this condition was understood.

Problem 63

Example of dividing 700 *loaves among four men in the proportion of the numbers* $\bar{\bar{3}}$, $\bar{2}$, $\bar{3}$ *and* $\bar{4}$. *Let me know the share that each man receives.*

Add $\bar{\bar{3}}$, $\bar{2}$, $\bar{3}$ and $\bar{4}$; it makes 1 $\bar{2}$ $\bar{4}$. Get 1 by operating on 1 $\bar{2}$ $\bar{4}$; it makes $\bar{2}$ $\bar{14}$. Take $\bar{2}$ $\bar{14}$ of 700; it is 400. $\bar{\bar{3}}$, $\bar{2}$, $\bar{3}$ and $\bar{4}$ of 400 will give the shares of the four men.

Do it thus:

The quantity is	700
$\bar{2}$ $\bar{14}$ of it is	400
$\bar{\bar{3}}$ of 400 to one is	266 $\bar{\bar{3}}$
$\bar{2}$ of 400 to one is	200
$\bar{3}$ of 400 to one is	133 $\bar{3}$
$\bar{4}$ of 400 to one is	100
Total	700.

Problem 64

Example of determining an arithmetical progression. Suppose it is said to thee, Distribute 10 hekat *of barley among* 10 *men in such a way that the shares shall be in arithmetical progression with a common difference of* $\frac{1}{8}$ hekat. *What is the share of each?*

The average share is 1 *hekat.* The number of differences will be 9, 1 less than the number of men. Take $\frac{1}{2}$ of the common difference; it makes $\frac{1}{16}$ *hekat.* Multiply this by 9; it makes $\frac{1}{2}$ $\frac{1}{16}$ *hekat.* Add it to the average share; 1 $\frac{1}{2}$ $\frac{1}{16}$ *hekat* is the largest share. Subtract the $\frac{1}{8}$ *hekat* for each man until you reach the last.

Do it thus:

The ten portions will be

1 $\frac{1}{2}$ $\frac{1}{16}$, 1 $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$, 1 $\frac{1}{4}$ $\frac{1}{16}$, 1 $\frac{1}{8}$ $\frac{1}{16}$, 1 $\frac{1}{16}$, $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$, $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{16}$, $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{16}$, $\frac{1}{2}$ $\frac{1}{16}$, $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$.

The total is 10 *hekat.*

This problem is explained in the Introduction, page 30. It will be seen from the Literal Translation that the author wrote "Example of distributing the differences," but the common difference is one of the numbers given in the statement of the problem, and the problem is to get the numbers in the progression itself.

Problem 65

Example of dividing 100 *loaves among* 10 *men, including a boatman, a foreman, and a door-keeper, who receive double portions. What is the share of each?*

The working out. Add to the number of the men 3 for those with double portions; it makes 13. Multiply 13 so as to get 100; the result is 7 $\frac{2}{3}$ $\frac{1}{39}$. This then is the ration for seven of the men, the boatman, the foreman, and the door-keeper receiving double portions.

For proof we add 7 $\frac{2}{3}$ $\frac{1}{39}$ taken 7 times and 15 $\frac{1}{3}$ $\frac{1}{26}$ $\frac{1}{78}$ taken 3 times for the boatman, the foreman, and the door-keeper. The total is 100.

Problem 66

If 10 hekat *of fat is given out for a year, what is the amount used in a day?*

The working out. Reduce the 10 *hekat* to *ro;* it makes 3200. Reduce the year to days; it makes 365. Get 3200 by operating on 365. The

result is 8 $\bar{3}$ $\overline{10}$ $\overline{2190}$. This makes for a day $\overline{64}$ *hekat* 3 $\bar{3}$ $\overline{10}$ $\overline{2190}$ *ro*.

Do it thus:

1	365
2	730
4	1460
8	2920
$\bar{3}$	243 $\bar{3}$
$\overline{10}$	36 $\bar{2}$
$\overline{2190}$	$\bar{6}$

Total 8 $\bar{3}$ $\overline{10}$ $\overline{2190}$.

Do the same thing in any example like this.

Here, as in Problem 61, our author generalizes, suggesting that the method can be applied in any case like this.

Problem 67

Example of reckoning the cattle of a herd. How many cattle are there in a herd when $\bar{3}$ of $\bar{3}$ of them make 70, *the number due as tribute to the owner?*

The herdsman came to the stock-taking with 70 cattle. The accountant said to the herdsman, Very few tribute-cattle art thou bringing; pray where are all thy tribute-cattle? The herdsman replied to him, What I have brought is $\bar{3}$ of $\bar{3}$ of the cattle that thou hast committed to me. Count and thou wilt find that I have brought the full number.

Do it thus:

1	1
$\bar{3}$	$\bar{3}$
$\bar{3}$	$\bar{3}$
$\bar{3}$ of $\bar{3}$	$\bar{6}$ $\overline{18}$.

Get 1 by operating on $\bar{6}$ $\overline{18}$.

1	$\bar{6}$ $\overline{18}$
2	$\bar{3}$ $\bar{9}$
\4	$\bar{3}$ $\bar{6}$ $\overline{18}$
\\$\bar{2}$	$\bar{9}$

Total 4 $\bar{2}$.

Multiply 70 by 4 $\bar{2}$; it makes 315. These are those committed to him.

Proof. Find $\frac{2}{3}$ of $\frac{1}{3}$ of 315.

1	315
$\frac{2}{3}$	210
$\frac{1}{3}$	105
$\frac{2}{3}$ of $\frac{1}{3}$	70.

These are those that he brought.

This problem indicates the method by which, when the herdsman brings to the owner or his accountant his tribute of cattle, the total count of the herd can be determined.

In the papyrus the author wrote, "Example of reckoning tribute," and though the tribute, 70 cattle, is then given, the conversation between the accountant and the herdsman implies that this is to be calculated from the number of the entire herd. But the calculations given are a solution of the inverse problem, to find the entire herd from the number of tribute-cattle, and only at the end, as a proof of these calculations, is the number of tribute-cattle determined from the number of the entire herd.

The problem is very similar to Problems 35–38, and the method of solution is the same as the method used in those problems. See Introduction, page 28.

Problem 68

Suppose a scribe says to thee, Four overseers have drawn 100 *great quadruple* hekat *of grain, their gangs consisting, respectively, of* 12, 8, 6 *and* 4 *men. How much does each overseer receive?*

There are 30 men in all. Multiply 30 so as to get 100; it makes 3 $\frac{1}{3}$. The amount given for each man is therefore 3 $\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{64}$ *hekat* 1 $\frac{2}{3}$ *ro*, Take this amount 12 times for the first overseer, 8 times for the second. 6 times for the third, and 4 times for the fourth.

The multiplication.

1	3 $\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{64}$ *hekat*	1 $\frac{2}{3}$ *ro*
2	6 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{32}$ "	3 $\frac{1}{3}$ "
4	13 $\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{64}$ "	1 $\frac{2}{3}$ "
8	26 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{32}$ "	3 $\frac{1}{3}$ "

List of the amounts of grain for the four overseers:

The first with 12 workmen will have			
$\frac{1}{4}$ of 100 *hekat* 15 *hekat*	or	40	*hekat*
The second with 8 workmen will have			
$\frac{1}{4}$ of 100 *hekat* 1 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{32}$ *hekat* 3 $\frac{1}{3}$ *ro*	"	26 $\frac{2}{3}$	"
The third with 6 workmen will have			
20 *hekat*	"	20	"

The fourth with 4 workmen will have
$13\ \frac{1}{4}\ \frac{1}{16}\ \frac{1}{64}$ *hekat* $1\frac{2}{3}$ *ro* or $13\ \frac{1}{3}$ *hekat*

The four with 30 workmen will have
100 *hekat* " 100 "

This problem is merely to divide 100 into four parts proportional to the numbers 12, 8, 6, and 4. The laborious numerical calculations are due to the use of the "Horus eye" fractions. See Introduction, pages 31–32.

The solution was lengthened in the papyrus also by much repetition. In particular, the multiplication of $3\ \frac{1}{4}\ \frac{1}{16}\ \frac{1}{64}$ *hekat* $1\ \frac{2}{3}$ *ro* was carried out four times, as far as 8 twice, and as far as 4 twice, the required products being checked and the total put down for each of the four overseers.

Problem 69 introduces the word *pefsu*, and Problems 69–78 are problems about *pefsu*. This word is derived from the stem of the verb "to cook." It meant something like "cooking ratio," that is, the number of units of food or drink that could be made from a unit of material in the process of cooking, and it determined the relative value of any food or drink. To preserve the technical sense I will use the word *pefsu* in my translation. We may note that the lower the *pefsu* the more valuable the unit of food.

The *pefsu* problems are very simple examples in arithmetic, made to seem more abstruse by the use of this word *pefsu*, and, in some of them, by the use of the word "exchange." We shall see that they can be stated in a way that will make explanations almost unnecessary.

Problem 69

$3\ \frac{1}{2}$ hekat *of meal is made into* 80 *loaves of bread. Let me know the amount of meal in each loaf and what is the* pefsu.

Multiply $3\ \frac{1}{2}$ so as to get 80.

1	$3\ \frac{1}{2}$
10	35
\20	70
\2	7
\\$\frac{2}{3}$	$2\ \frac{1}{3}$
\\$\frac{1}{21}$	$\frac{1}{6}$
\\$\frac{1}{7}$	$\frac{1}{2}$.

The *pefsu* is $22\ \frac{2}{3}\ \frac{1}{7}\ \frac{1}{21}$.

Proof.

\1	22 $\frac{2}{3}$ $\frac{1}{7}$ $\frac{1}{21}$
\2	45 $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{14}$ $\frac{1}{28}$ $\frac{1}{42}$
\\$\frac{1}{2}$	11 $\frac{1}{3}$ $\frac{1}{14}$ $\frac{1}{42}$
Total	80.

3 $\frac{1}{2}$ *hekat* makes 1120 *ro*, for

\1	320
\2	640
\\$\frac{1}{2}$	160
Total	1120.

Therefore multiply 80 so as to get 1120.

Do it thus:

1	80
\10	800
2	160
\4	320
Total 14.	

That is, one of the loaves contains 14 *ro*, or $\frac{1}{32}$ *hekat* 4 *ro*, of meal.

Proof.

1	$\frac{1}{32}$	*hekat*	4 *ro*
2	$\frac{1}{16}$ $\frac{1}{64}$	"	3 "
4	$\frac{1}{8}$ $\frac{1}{32}$ $\frac{1}{64}$	"	1 "
8	$\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{32}$	"	2 "
\16	$\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{16}$	"	4 "
32	1 $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{64}$	"	3 "
\64	2 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{32}$ $\frac{1}{64}$	"	1 "

It makes 3 $\frac{1}{2}$ *hekat* of meal for the 80 loaves.

The author asks two questions, first, what is the amount of meal in 1 loaf of bread, and second, how many loaves of bread one *hekat* of meal will make. In the solution, however, he determines first the number of loaves that one *hekat* will make, which he finds to be 22 $\frac{2}{3}$ $\frac{1}{7}$ $\frac{1}{21}$. This is the *pefsu*. Before determining the amount of meal in one loaf he reduces 3 $\frac{1}{2}$ *hekat* to 1120 *ro*, so that he can say, If 80 loaves take 1120 *ro*, 1 loaf will take 14 *ro*, for 80 times 14, or, what is the same thing, 14 times 80 makes 1120. The 14 *ro* is written in standard from as $\frac{1}{32}$ *hekat* 4 *ro*, and this is the answer to the first question.

In the first multiplication it would seem as if the easiest way to get $\frac{1}{21}$ would be from $\frac{1}{7}$ by halving and taking $\frac{2}{3}$. Perhaps originally it was obtained in that way. In the second line of the first proof there are two applications of the table at the beginning of the papyrus, the double of $\frac{1}{7}$ being $\frac{1}{4}$ $\frac{1}{28}$ and the double of $\frac{1}{21}$, $\frac{1}{14}$ $\frac{1}{42}$.

Problem 70

7 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ hekat *of meal is made into* 100 *loaves of bread. What is the amount of meal in each loaf and what is the* pefsu?

Multiply 7 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ so as to get 100.

1	7 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$
2	15 $\frac{1}{2}$ $\frac{1}{4}$
\4	31 $\frac{1}{2}$
\8	63
\\$\frac{2}{3}$	5 $\frac{1}{4}$
Total	99 $\frac{1}{2}$ $\frac{1}{4}$
Remainder	$\frac{1}{4}$
$\frac{1}{63}$	$\frac{1}{8}$.

Double the fraction for $\frac{1}{4}$

\\$\frac{1}{42}$ $\frac{1}{126}$	$\frac{1}{4}$.

The *pefsu* is 12 $\frac{2}{3}$ $\frac{1}{42}$ $\frac{1}{126}$.

Proof.

\1	12 $\frac{2}{3}$ $\frac{1}{42}$ $\frac{1}{126}$
\2	25 $\frac{1}{3}$ $\frac{1}{21}$ $\frac{1}{63}$
\4	50 $\frac{2}{3}$ $\frac{1}{14}$ $\frac{1}{21}$ $\frac{1}{126}$
\\$\frac{1}{2}$	6 $\frac{1}{3}$ $\frac{1}{84}$ $\frac{1}{252}$
\\$\frac{1}{4}$	3 $\frac{1}{6}$ $\frac{1}{168}$ $\frac{1}{504}$
\\$\frac{1}{8}$	1 $\frac{1}{2}$ $\frac{1}{12}$ $\frac{1}{336}$ $\frac{1}{1008}$
Total	100.

7 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ *hekat* make 2520 *ro*, for

1	320
2	640
4	1280
$\frac{1}{2}$	160
$\frac{1}{4}$	80
$\frac{1}{8}$	40
Total	2520.

Therefore multiply 100 so as to get 2520.

1	100
10	1000
\20	2000
\5	500
\\$\frac{1}{5}$	20

Total 25 $\frac{1}{5}$.

That is, one of the loaves contains 25 $\frac{1}{5}$ *ro*, or $\frac{1}{16}$ $\frac{1}{64}$ *hekat* $\frac{1}{5}$ *ro*.

Proof.

1	$\frac{1}{16}$ $\frac{1}{64}$	*hekat*	$\frac{1}{5}$	*ro*
10	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{32}$	"	2	"
100	$7\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$	"		

This problem is similar to the preceding. The reduction of the given number of *hekat* to *ro* is omitted in the papyrus, but the result, 2520, is given, and given as if it were the result of the multiplication used to prove the first part of the solution. Evidently, in copying this solution, the scribe let his eye drop from one "Total" to the next, and so left out the result of one multiplication and all the partial products of the next (Peet, page 116). See note to Problem 28.

There are some interesting steps in the first multiplication and in the first proof. We may notice also that in the first multiplication, as also in Problem 69, we have the "Horus eye" fractions given in the statement of the problem written as ordinary fractions. See Introduction, page 31, footnote 3.

Problem 71

From 1 des-*measure of beer* $\frac{1}{4}$ *has been poured off, and then the measure has been filled up with water. What is the* pefsu *of the diluted beer?*

Reckon the amount of *besha*[1] in 1 *des* of beer; it is $\frac{1}{2}$ *hekat*. Take away $\frac{1}{4}$ of it, namely $\frac{1}{8}$ *hekat;* the remainder is $\frac{1}{4}$ $\frac{1}{8}$ *hekat*. Multiply $\frac{1}{4}$ $\frac{1}{8}$ *hekat* so as to get 1 *hekat*; the result is 2 $\frac{2}{3}$, and this is the *pefsu*.

The solution as given means that when *besha* is made into beer, $\frac{1}{2}$ *hekat* of *besha* will make 1 *des*-jug of beer. If the beer is made weaker by pouring off $\frac{1}{4}$ of it and filling with water, it will take less *besha* by $\frac{1}{4}$ to make enough of this diluted beer to fill the jug, namely, $\frac{1}{4}$ $\frac{1}{8}$ *hekat*, and so 1 *hekat* of *besha* will make 2 $\frac{2}{3}$ *des* of this weaker kind of beer. *Pefsu* then means here the number of *des*-jugs of beer that a *hekat* of *besha* will make, just as in the preceding two problems it means the number of loaves that a *hekat* of meal will make.

The first $\frac{1}{2}$ *hekat* in the solution is written by mistake in the papyrus as an ordinary $\frac{1}{2}$. Elsewhere the fractions of a *hekat* are written in the "Horus eye" form.

The mention of *besha* and of other kinds of food may tend to make these solutions seem more obscure, but in no way do they have anything to do with the reasoning or numerical work, and we can leave them out of consideration in trying to understand this.

Problem 72

Example of exchanging loaves for other loaves. Suppose it is said to thee, 100 *loaves of* pefsu 10 *are to be exchanged for a number of loaves of* pefsu 45. *How many of these will there be?*

[1] For a list of kinds of grain or food mentioned in the papyrus see Introduction, page 46.

Find the excess of 45 over 10; it is 35. Multiply 10 so as to get 35; it makes 3 ½. Multiply 100 by 3 ½; it makes 350. Add 100 thereto; it makes 450. Say then that there are exchanged 100 loaves of *pefsu* 10 for 450 loaves of *pefsu* 45, making in *wedyet*-flour 10 *hekat*.

In this Problem our author asks the question, If one has 100 loaves with a *pefsu* of 10, for how many loaves with a *pefsu* of 45 can he exchange them? He arrives at the result in a round-about way. He could simply have determined that 10 *hekat*, the amount of meal required to produce the given 100 loaves, would have produced 10 times 45 loaves of the second kind, and he would thus have arrived at the same result. This is the way in which he solves the next problem and Problem 75, both of which are problems of the same kind.

Problem 73

Suppose it is said to thee, 100 *loaves of* pefsu 10 *are to be exchanged for loaves of* pefsu 15. *How many of these will there be?*

Reckon the amount of *wedyet*-flour in these 100 loaves; it is 10 *hekat*. Multiply 10 by 15; it makes 150. This then is the number of loaves for the exchange.

Do it thus: 100 loaves of *pefsu* 10 would be exchanged for 150 loaves of *pefsu* 15. It takes 10 *hekat*.

As in Problem 72, 100 loaves of *pefsu* 10 require 10 *hekat* of *wedyet*-flour, for "*pefsu* 10" means that 1 *hekat* makes 10 loaves. Now in this problem a *hekat* will make 15 loaves of the second kind, and so 10 *hekat* will make 150 loaves. Therefore 150 loaves of the second kind will be the equivalent of the 100 given loaves of the first kind.

Problem 74

Another problem. 1000 *loaves of* pefsu 5 *are to be exchanged, a half for loaves of* pefsu 10, *and a half for loaves of* pefsu 20. *How many of each will there be?*

1000 loaves of *pefsu* 5 will take 200 *hekat* of Upper Egyptian barley. This then is the amount of *wedyet*-flour in these loaves. ½ of the 200 *hekat* is 100 *hekat*. Multiply 100 by 10; it makes 1000, the number of loaves of *pefsu* 10. Multiply 100 by 20; it makes 2000, the number of loaves of *pefsu* 20.

Do it thus: 1000 loaves of *pefsu* 5, made from 200 *hekat* of *wedyet*-flour, can be exchanged for 1000 loaves of *pefsu* 10, taking 100 *hekat*, and 2000 loaves of *pefsu* 20, taking 100 *hekat*.

In this problem the given 1000 loaves require 200 *hekat* of *wedyet*-flour. Then, if the same amount is used to make two kinds of loaves, one-half of it at the rate of 10 loaves to a *hekat*, and the other half at the rate of 20 loaves to a *hekat*, the result will be 1000 loaves of the first kind and 2000 loaves of the second kind.

Problem 75

Another problem. 155 *loaves of* pefsu 20 *are to be exchanged for loaves of* pefsu 30. *How many of these will there be?*

The amount of *wedyet*-flour in the 155 loaves of *pefsu* 20 is 7 $\frac{1}{2}$ $\frac{1}{4}$ *hekat.* Multiply this by 30; it makes 232 $\frac{1}{2}$.

Do it thus: 155 loaves of *pefsu* 20, made from 7 $\frac{1}{2}$ $\frac{1}{4}$ *hekat* of *wedyet*-flour, can be exchanged for 232 $\frac{1}{2}$ loaves of *pefsu* 30. It takes 7 $\frac{1}{2}$ $\frac{1}{4}$ *hekat.*

This problem is like Problems 72 and 73, and the solution given is like that for Problem 73.

Problem 76

Another problem. 1000 *loaves of* pefsu 10 *are to be exchanged for a number of loaves of* pefsu 20 *and the same number of* pefsu 30. *How many of each kind will there be?*

One loaf of each kind will take

$\frac{1}{20}$ and $\frac{1}{30}$ of a *hekat.*

As parts of 30 these are

1 $\frac{1}{2}$ and 1, together 2 $\frac{1}{2}$.

Multiply 2 $\frac{1}{2}$ so as to get 30

1	2 $\frac{1}{2}$
\10	25
\2	5
Total 12.	

Therefore 2 $\frac{1}{2}$ is $\frac{1}{12}$ of 30, so that $\frac{1}{20}$ $\frac{1}{30}$ equals $\frac{1}{12}$. Two loaves, one of each kind, will take $\frac{1}{12}$ of a *hekat* and 1 *hekat* will make 12 loaves of each kind.

The quantity of *wedyet*-flour in the 1000 loaves is 100 *hekat.* Multiply 100 by 12; the result is 1200, which is the number of loaves of each kind for the exchange. That is

1000 loaves of *pefsu* 10, making in *wedyet*-flour 100 *hekat* can be exchanged for

1200 loaves of *pefsu* 20,	"	"	"	$\frac{1}{2}$ of "	10 *hekat*
and 1200 " " " 30	"	"	"	$\frac{1}{4}$ of "	15 "

In this problem, as in Problem 74, the author wishes to know how many loaves of two kinds can be made from a certain amount of *wedyet*-flour, but in Problem 74 he uses half of the flour for one kind of loaf and half for the other. This time he wishes to

make the same number of loaves of each kind, which makes the problem typical of a very interesting class of problems. See Introduction, page 29.

The amount given is the amount required to make 1000 loaves with a *pefsu* of 10; that is, it is 100 *hekat*. The two kinds of loaves that he wishes to get are loaves with a *pefsu* of 20 and loaves with a *pefsu* of 30. One loaf of the first kind will take $\frac{1}{20}$ of a *hekat* and one loaf of the second kind will take $\frac{1}{30}$ of a *hekat*. He applies these fractions to 30. Perhaps he thinks of determining how much flour it will take to make 30 loaves of each kind, and this he finds to be 2 $\frac{1}{2}$ *hekat*. Now as many times as 2 $\frac{1}{2}$ must be multiplied to produce 30, namely, 12 times, 1 *hekat* making 12 loaves of each kind, so many times must 100 be multiplied to produce the number of loaves of each kind that 100 *hekat* will make.

At the end in what appears to be a proof he puts down the amount of *wedyet*-flour that 1200 loaves of each kind will require, showing that it makes just 100 *hekat*. We may notice the form in which he expresses these amounts, writing the quarter and half of 100 *hekat* and the number of *hekat* in addition. This is the form used for large quantities when he would use the "Horus eye" fractions for small quantities. The same forms are used in Problem 68 and in some of the problems that are to follow. See Introduction, pages 31-32.

Problem 77

Example of exchanging beer for bread. Suppose it is said to thee, 10 des *of beer (of* pefsu 2) *are to be exchanged for loaves of* pefsu 5. *How many loaves will there be?*

Reckon the amount of *wedyet*-flour in 10 *des* of beer; it is 5 *hekat*. Multiply 5 by 5; it makes 25. Say then that it takes 25 loaves to make the exchange.

Do it thus:

10 *des* of beer taking 5 *hekat* of *wedyet*-flour
can be exchanged for
25 loaves of bread of *pefsu* 5; for these also take 5 *hekat* of *wedyet*-flour.

Problem 78

Example of exchanging bread for beer. Suppose it is said to thee, 100 *loaves of* pefsu 10 *are to be exchanged for a quantity of beer of* pefsu 2. *How many* des *of beer will there be?*

Reckon the amount of *wedyet*-flour in 100 loaves of *pefsu* 10; it is 10 *hekat*. Multiply 10 by 2; it makes 20. Say then that it takes 20 *des* of beer to make the exchange.

These two problems are like Problems 72, 73 and 75, but instead of two kinds of loaves we have here an exchange of loaves and beer.

Problem 79

Sum the geometrical progression of five terms, of which the first term is 7 and the multiplier 7.

The sum according to the rule. Multiply 2801 by 7.

1	2801
2	5602
4	11204
Total	19607

The sum by addition.

houses	7
cats	49
mice	343
spelt	2401
hekat	16807
Total	19607.

I have discussed the numerical work of this problem in the Introduction, page 30.

In the second column the author places before the successive powers of 7 the words, houses, cats, mice, spelt and *hekat*. Eisenlohr regards these words as names given to the powers of 7. Another interpretation is that the problem intended is like this: In each of 7 houses are 7 cats, each cat kills 7 mice, each mouse would have eaten 7 ears of spelt, and each ear of spelt will produce 7 *hekat* of grain; how much grain is thereby saved? But the author adds all of these quantities together, showing that he is more interested in the numerical problem of the sum of these numbers.

This interpretation, coupled with the number 7, reminds us of the children's rhyme, of which one version is the following:

"As I was going to Saint Ives,
I met a man with seven wives.
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits;
Kits, cats, sacks and wives,
How many were there going to Saint Ives?" [1]

Here, again, it is suggested that the sum total of a geometrical progression be calculated but there is a "joker" in the actual wording of the first and last lines.

Rodet (1882, page 111) found in the *Liber Abaci* of Leonardo of Pisa (see Bibliography, 1857) a problem of a geometrical progression expressed in much the same way, and having the ratio 7, and he suggests that Problem 79, absurd as is its heterogeneous addition, has perpetuated itself through all the centuries from the times of the ancient Egyptians.

[1] *Every Child's Mother Goose*, with Introduction by Carolyn Wells, New York, 1918, page 111.

Problem 80

Express the "Horus eye" fractions in terms of the hînu.

The following vessels are used[1] in measuring grain by the functionaries of the granary:

1	*hekat*	makes	10	*hînu*
½	"	"	5	"
¼	"	"	2 ½	"
⅛	"	"	1 ¼	"
1/16	"	"	½ ⅛	"
1/32	"	"	¼ 1/16	"
1/64	"	"	⅛ 1/32	"

Problem 81

Another reckoning. Express fractions of a hekat *as "Horus eye" fractions and in terms of the* hînu.

Now	½	*hekat*	makes	5	*hînu*
	¼	"	"	2 ½	"
	⅛	"	"	1 ¼	"
	1/16	"	"	½ ⅛	"
	1/32	"	"	¼ 1/16	"
	1/64	"	"	⅛ 1/32	"

a Now

½ ¼ ⅛	*hekat*		makes	8 ½ ¼	*hînu*			
½ ¼	"		"	7 ½	"			
½ ⅛ 1/32	"	3 ⅓ *ro*	"	6 ⅔	"	it is	⅔	of a *hekat*
½ ⅛	"		"	6 ¼	"	" "	½ ⅛	" " "
¼ ⅛	"		"	3 ½ ¼	"	" "	¼ ⅛	" " "
¼ 1/16 1/64	"	1 ⅔ "	"	3 ⅓	"	" "	⅓	" " "
¼	"		"	2 ½	"	" "	¼	" " "
⅛ 1/16	"	4 "	"	2	"	" "	⅙	" " "
⅛ 1/32	"	3 ⅓ "	"	1 ⅔	"	" "	⅙	" " "

b Now

⅛ 1/16	"	4 "	"	2	"	" "	⅙	" " "
1/16 1/32	"	2 "	"	1	"	" "	1/10	" " "
1/32 1/64	"	1 "	"	½	"	" "	1/20	" " "
1/64	"	3 "	"	¼	"	" "	1/40	" " "
1/16	"	1 ⅓ "	"	⅔	"	" "	1/15	" " "

[1] This interpretation is suggested by Gunn (page 136).

c							
1/32	*hekat*	2/3 *ro*	makes	1/3	*hînu*	it is 1/30	of a *hekat*
1/64	"	1/3 "	"	1/6	"	" " 1/60	" " "
1/2	"		"	5	"	" " 1/2	" " "
1/4	"		"	2 1/2	"	" " 1/4	" " "
1/2 1/4	"		"	7 1/2	"	" " 1/2 1/4	" " "
1/2 1/4 1/8	"		"	8 1/2 1/4	"	" " 1/2 1/4 1/8	" " "
d							
1/2 1/8	"		"	6 1/4	"	" " 1/2 1/8	" " "
1/4 1/8	"		"	3 1/2 1/4	"	" " 1/4 1/8	" " "
1/2 1/8 1/32	"	3 1/3 "	"	6 2/3	"	" " 2/3	" " "
1/4 1/16 1/64	"	1 2/3 "	"	3 1/3	"	" " 1/3	" " "
1/8	"		"	1 1/4	"	" " 1/8	" " "
1/16	"		"	1/2 1/8	"	" " 1/16	" " "
e							
1/32	"		"	1/4 1/16	"	" " 1/32	" " "
1/64	"		"	1/8 1/32	"	" " 1/64	" " "

This problem is interesting since it gives a very full table of fractional parts of a *hekat* in terms of "Horus eye" fractions and in terms of the *hînu.* There are a few errors in the papyrus and in some places it is difficult to determine just what the author intended to write. In the translation given above I have made the corrections that seem most probable.

Except for the first section, which is a repetition of Problem 80, the table is in three columns. The first column gives an expression in "Horus eye" fractions, the second column in terms of the *hînu* [1] and the third (in red in the papyrus) as a simple part of a *hekat.* In section *a* this third column was placed before the other two and written in black. Peet has suggested (page 124) that the scribe, writing the black portions first, forgot to leave room for this first red section in its proper place.

In the fourth and fifth lines of section *a* the quantities expressed are 1/2 1/8 and 1/4 1/8 *hekat,* equal to 5 and 3 times 1/8 *hekat.* In the third column (placed in the papyrus before the other two) the author wrote 1/5 and 3. If 1/5 were a mistake for 5, a superfluous dot being inserted, we might suppose that he thought of 1/8 *hekat* as a kind of unit of which he takes 5 and 3, respectively. The expressions in these two lines are given correctly as lines 1 and 2 of section *d.*

The sixth line of section *a,* as written in the papyrus, is the most confusing. The best explanation seems to be that the author intended to write 1/3 of a *hekat,* getting this line by halving the quantities in the third line, although he says 1/5. In the *hînu* column he wrote for 3 1/3, 3 1/4 1/8 2/3, and in the third line for 6 2/3, 6 1/2 1/16. In a large part of the table the fractions of a *hînu* happen to be fractions whose denominators are powers of 2 like the "Horus eye" fractions and it may be that he attempted to write the fractions in the third and sixth lines in this way. In lines 3 and 4 of section *d* he gives correctly

[1] It may be noted that the fractions of a *hînu* are not written like the "Horus eye" fractions, even when the denominators are powers of 2.

the expressions for $\frac{1}{8}$ and $\frac{1}{4}$ of a *hekat*, and in the above table I have corrected lines 3 and 6 of section *a* so that they are the same as lines 3 and 4 of section *d*.

In the first line of section *c*, at the end, the papyrus says $\frac{1}{60}$ of a *hekat* when it should be $\frac{1}{30}$, and if the amount in the second line is intended to be a half of the amount in the first line, as is probable, then the middle number should be $\frac{1}{6}$, and the last number $\frac{1}{60}$, instead of $\frac{1}{3}$ and $\frac{1}{30}$, as in the papyrus. In the sixth line of the same section the fraction $\frac{1}{4}$ is omitted in the middle number, and a little further on, in the second line of section *d*, the 3 is omitted from the middle number. I have corrected all of these mistakes in the table above.

Problems 82–84 have to do with the feed of birds and oxen and are interesting chiefly as showing with how much care the Egyptian farmer estimated the amount of feed necessary for each bird or animal. There are many mistakes in these problems, and the last one, Problem 84, is unintelligible; at least no one has yet explained it.

Problem 82

Estimate in wedyet-*flour, made into bread, the daily portion of feed for geese.*

10 fatted geese eat daily	$2\frac{1}{2}$ *hekat*,
Taking for 10 days	$\frac{1}{4}$ of 100 *hekat*;
Taking for 40 days	100 *hekat*.

The amount of spelt that has to be ground to produce it is

1 $\frac{2}{3}$ times 100 *hekat* 16 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{32}$ *hekat* 3 $\frac{1}{3}$ *ro*.

The amount of wheat is

$\frac{1}{8}$ $\frac{1}{4}$ of 100 *hekat* 8 $\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{64}$ *hekat* 1 $\frac{2}{3}$ *ro*.

That which has to be taken away is $\frac{1}{10}$, of this, namely,

6 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{32}$ *hekat* 3 $\frac{1}{3}$ *ro*.

The remainder, which is the amount of grain required, is

$\frac{1}{2}$ $\frac{1}{4}$ of 100 *hekat* 18 $\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{64}$ *hekat* 1 $\frac{2}{3}$ *ro*.

Expressed in double *hekat* this is

$\frac{1}{4}$ of 100 *hekat* 21 $\frac{1}{2}$ $\frac{1}{8}$ $\frac{1}{32}$ *hekat* 3 $\frac{1}{3}$ *ro*.

Problem 82B

Estimate the amount of feed for other geese.

If to fatten 10 geese it takes daily	1 $\frac{1}{4}$ *hekat*,
It will take for 10 days	12 $\frac{1}{2}$ "
And for 40 days	$\frac{1}{2}$ of 100 *hekat*

The amount of grain to be ground in double *hekat* is

23 $\frac{1}{4}$ $\frac{1}{16}$ $\frac{1}{64}$ *hekat* 1 $\frac{2}{3}$ *ro*.

These are problems, about the feed for fattening geese. First the author determines that it takes 100 *hekat* to feed 10 fattening geese for 40 days. Then he asks how much grain is required to produce in grinding 100 *hekat*.

He begins by mentioning two kinds of grain. The first appears to be spelt. The second is wheat. Adding to 100 *hekat* $\frac{2}{3}$ of 100 *hekat*, which would make 166 $\frac{2}{3}$ *hekat*, he says that this is the amount of spelt that has to be ground to produce 100 *hekat* of meal. Then he simply takes $\frac{2}{3}$ of 100 *hekat*, namely, 66 $\frac{2}{3}$ *hekat*, and says that this is the amount of wheat that would be required. In the next line he takes $\frac{1}{10}$ of the latter quantity, says that this is the amount to be taken away, and subtracts it from 100 *hekat*, presenting the remainder as the solution of the problem. In other words, whatever the two lines about spelt and wheat may mean, we may suppose that grinding increases the bulk of the grain (wheat perhaps), and that the amount required to produce 100 *hekat* of meal was to be determined by taking away from 100 *hekat* of grain $\frac{1}{10}$ of $\frac{2}{3}$ of it. This may have been a rule that had been established by experiment and was well-known, or the author may have made up an empirical rule to determine somewhat roughly the smaller amount of grain that he knew would be sufficient to make a given amount of meal. See notes to Problem 53, page 94.

In Problem 82B, which Eisenlohr included in the previous problem, we have a similar series of calculations for geese, assuming this time that it takes just half as much to fatten them. All the numbers are half as large as in Problem 82, and so all of the steps are omitted and only the last line is given.

The quantities in these solutions are expressed in the form which uses the "Horus eye" fractions of a *hekat*, and for larger quantities writes the number of times 100 *hekat*, and writes 50 *hekat* and 25 *hekat* as $\frac{1}{2}$ and $\frac{1}{4}$ of 100 *hekat*. See the notes to Problem 76, page 111, and Introduction, pages 31–32. In this notation $\frac{2}{3}$ of 100 *hekat*, or 66 $\frac{2}{3}$ *hekat* would be written $\frac{1}{2}$ of 100 *hekat* 16 $\frac{1}{2}\,\frac{1}{8}\,\frac{1}{32}$ *hekat* 3 $\frac{1}{3}$ *ro*, and this is the way in which the author writes it first, in the expression for 1 $\frac{2}{3}$ times 100 *hekat*, but in the next expression he writes $\frac{1}{3}$, and then for the other third just a half of these numbers. I am inclined to think that the $\frac{1}{3}$ was a little irregular, and that the scribe became confused in trying to write down $\frac{2}{3}$. $\frac{1}{10}$ of $\frac{2}{3}$ of 100 *hekat*, or 6 $\frac{2}{3}$ *hekat* is written correctly in the papyrus. 6 $\frac{2}{3}$ from 100 leaves 93 $\frac{1}{3}$, and 93 $\frac{1}{3}$ *hekat* reduces without difficulty to the expression given. We can perform these various operations directly with the forms used by the Egyptian.

We may notice that we have in this problem the double *hekat* and its parts with the double *ro*. The double *hekat* is also mentioned in Problem 84. See Introduction, page 32.

Problem 83

Estimate the feed necessary for various kinds of birds.

If the feed of four geese that are cooped up is 1 *hînu* of Lower-Egyptian grain, the portion of one of the geese is $\frac{1}{64}$ *hekat* 3 *ro*.

If the feed of a goose that goes into the pond is $\frac{1}{16}\,\frac{1}{32}$ *hekat* 2 *ro*, it is 1 *hînu* for 1 goose.

For 10 geese it takes 1 *hekat* of Lower-Egyptian grain.

For 10 days 10 *hekat*.

For a month $\frac{1}{4}$ of 100 *hekat* 5 *hekat*.

The daily portion of feed to fatten

A goose	is 1/8 1/32 *hekat*	3 1/3 *ro*	for 1 bird
A *terp*-goose	" 1/8 1/32 "	3 1/3 "	" " "
A crane	" 1/8 1/32 "	3 1/3 "	" " "
A *set*-duck	" 1/32 1/64 "	1 "	" " "
A *ser*-goose	" 1/64 "	3 "	" " "
A dove	"	3 "	" " "
A quail	"	3 "	" " "

In regard to the identification of these birds see Peet, page 126.

Problem 84

Estimate the feed of a stall of oxen.

	. . . food	common . . . food
4 fine Upper Egyptian bulls eat	24 *hekat*	2 *hekat*
2 fine Upper Egyptian bulls eat	22 "	6 "
3 common . . . cattle eat	20 "	2 "
1 . . . ox	20 "	
Total of this feed	86 "	10 "
It makes in spelt	9 "	7 1/2 "
It makes for 10 days	1/2 1/4 of 100 *hekat* 15 *hekat*	1/2 1/4 of 100 *hekat*
It makes for a month	200 *hekat*	1/2 1/4 of 100 *hekat* 15 *hekat*
It makes in double *hekat*	1/2 of 100 *hekat* 11 1/2 1/8 *hekat* 3 *ro*	1/4 of 100 *hekat* 5 *hekat*

Eisenlohr gave the numbers 85, 86, and 87 to certain fragments that are not a part of the mathematical work of the papyrus, but are of interest, although incomplete and more or less unintelligible. As they cannot in any sense be called problems I will use the word "Number" when referring to them, but will give them these numbers, as does Eisenlohr, in continuation of the numbering of the problems.[1]

[1] Strictly Number 87 should be numbered before Number 86, which is at the very end of the papyrus. See Diagram in the second volume.

Number 85

This is a group of cursive hieroglyphic signs, written upside down on the back of the papyrus. It has been suggested that the scribe was merely trying his pen (see Peet, page 128). Eisenlohr attempted to give a meaning to the group, calling it a "Motto," and translating it somewhat as follows: Kill vermin, mice, fresh weeds, numerous spiders. Pray the god Rê' for warmth, wind and high water.

Gunn, however, claims (page 136) that we have here an early example of the so-called enigmatic writing, and gives as a tentative translation, "Interpret this strange matter, which the scribe wrote . . . according to what he knew."

Numbers 86 and 87 are pieces from some other writings pasted on the back of the papyrus to strengthen it or to mend places where it was torn. Number 86 is upside down. My translation of these fragments follows chiefly that of Peet.

Number 86

This seems to be from some account or memorandum. There are three pieces which appear separated in the British Museum Facsimile, but since that was made they have been placed together in their proper relative positions as may be seen in Photograph 31, volume 2. There are eighteen lines, but parts are missing from both ends of the lines. The following is a translation of the words that remain in the eighteen lines:

1\. . . . living forever. List of the food in Hebenti . . .
2\. . . . his brother the steward Ka-mosĕ . . .
3\. . . . of his year, silver, 50 pieces twice in the year . . .
4\. . . . cattle 2, in silver 3 pieces in the year . . .
5\. . . . one twice; that is, $\frac{1}{6}$ and $\frac{1}{6}$. Now as for one . . .
6\. . . . 12 *hînu;* that is, silver, $\frac{1}{4}$ piece; one . . .
7\. . . . (gold or silver) 5 pieces, their price therefor; fish, 120, twice . . .
8\. . . . year, barley, in quadruple *hekat,* $\frac{1}{2}$ $\frac{1}{4}$ of 100 *hekat* 15 *hekat;* spelt, 100 *hekat* . . . *hekat* . . .
9\. . . . barley, in quadruple *hekat,* $\frac{1}{2}$ $\frac{1}{4}$ of 100 *hekat* 15 *hekat;* spelt, 1 $\frac{1}{2}$ $\frac{1}{4}$ times 100 *hekat* 17 *hekat* . . .
10\. . . . 146 $\frac{1}{2}$; barley, 1 $\frac{1}{2}$ $\frac{1}{4}$ times 100 *hekat* 10 *hekat*; spelt, 300 *hekat* . . . *hekat* . . .
11\. . . . $\frac{1}{2}$, there was brought wine, 1 ass(-load?) . . .
12\. . . . silver $\frac{1}{2}$ piece; . . . 4; that is, in silver . . .
13\. . . . 1 $\frac{1}{4}$; fat, 36 *hînu;* that is, in silver . . .
14\. . . . 1 $\frac{1}{2}$ $\frac{1}{4}$ times 100 *hekat* 21 *hekat*; spelt, in quadruple *hekat,* 400 *hekat* 10 *hekat* . . .
15–18. [These lines are repetitions of line 14.]

Number 87

This seems to be a memorandum of some incidents, not very coherent, but apparently complete.

Year 11, second month of the harvest season, Heliopolis was entered.

The first month of the inundation season, 23rd day, the commander (?) of the army (?) attacked (?) Zaru.

25th day, it was heard that Zaru was entered.

Year 11, first month of the inundation season, third day, Birth of Set; the majesty of this god caused his voice to be heard.

Birth of Isis, the heavens rained.

See the note on the Egyptian calendar and Egyptian chronology, Introduction, page 43.

CHRONOLOGICAL LIST OF DOCUMENTS DISCUSSED

In more than one case the dates assigned below can be only very approximate; for example, since the Akhmîm papyrus is probably not earlier than 600, nor later than 900, the year half way between these years has been given. When printed sources gave no dates the leading living authorities were consulted for determining them. An attempt has been made to adjust to recent scholarship the dates in printed sources.

Approximate Dates	*Documents*	*Locations*
B. C.		
3500	Hierakonopolis mace (hieroglyphic)	Oxford
2200	Babylonian tablet 10201 from Nippur[1] (?) (cuneiform)	Philadelphia
2200	Tello tablet survey (cuneiform)	Constantinople
2150	Babylonian tablet 12648 from Nippur (?) (cuneiform)	Philadelphia
2070	Babylonian tablets (25) from Nippur (?) (cuneiform)	Philadelphia
2000	Akkadian clay tablet (cuneiform)	Berlin (?)
2000	Akhmîm tablets 25367, 25368 (hieratic)	Cairo
2000	Geometrical tablet 15285 (cuneiform)	London
2000	Mathematical tablets 85194, 85210 (cuneiform) . . .	London
1900	Babylonian tablet CBS 8536 (cuneiform)	Philadelphia
1900	Book of the Dead, Introduction to Section 99, Berlin 9 and Cairo 28023 (hieratic)	Berlin and Cairo
1900	Kahun counting stick 372–59 (hieroglyphic)	London
1850	Golenishchev mathematical papyrus (hieratic) . . .	Moscow
1850	Kahun papyri (hieratic)	London
1850	Thebes papyrus 6619 (hieratic)	Berlin
1800	Babylonian tablet 19797 from Nippur[2] (?) (cuneiform)	Philadelphia
1700	Bulak papyrus 11, 18 (hieratic)	Cairo
1650	Leather roll 10250[3] (hieratic)	London
1650	Rhind mathematical papyrus 10057, 10058 (hieratic) .	London
1650	Rhind papyrus fragments 265 to go between 10057, 10058, above (hieratic)	New York
1650	Tablet 7798 (hieroglyphic)	Berlin
1465	Papyri 9784, 9785 (hieratic)	Berlin

[1] A question mark is here added because many assyriologists are in doubt as to whether some tablets, claimed by Hilprecht to have come from Nippur, were ever there.

[2] Transcribed but not interpreted.

[3] Wholly unpublished.

Approximate Dates	*Documents*	*Locations*
B. C.		
1400	Karnak water clock (hieroglyphic)	Cairo
1350	Babylonian tablets (22) from Nippur (?) (cuneiform)	Philadelphia
1350	Rollin papyri (hieratic)	Paris
1315	Anastasy papyrus I 350 (hieratic)	Leyden
1000	Temple inscription, an ellipse	Luxor
280 to 90	Papyri XVI, XX, XXII, XXVII, XXVIII, XXX, XXXVII, XXXVIII, XL (demotic)	Manchester
250	Elephantine ostraca P 11999, P 12000, P 12002, P 12007, P 12008 (Greek)[1]	Berlin
210	Bilingual papyrus contract sale 10463 (demotic and Greek)	London
175	Tebtunis papyrus 87 (Greek)	Oxford[2]
100	Temple inscriptions (hieroglyphic)	Edfu
A. D.		
100	Papyrus CCLXVII (Greek)	London
140	Papyrus table of fractions (demotic)	London
150	Ayer papyrus (Greek)	Chicago
150	Papyrus 11529 (Greek)	Berlin
250	Oxyrhynchus papyrus 470 (Greek)	Dublin
350	Byzantine tablet of fractions 374–92 (Greek)	London
350	Michigan mathematical papyrus 621 (Greek)	Ann Arbor
350	Oxyrhynchus papyrus 186 (Greek)	Florence
350	Papyrus Gr 19996 (Greek)[1]	Vienna
550	Ostracon 480 (Coptic)	London
550	Ostracon 29750 (Coptic)	London
600	Papyrus 2241, nos. 22–28 (Greek)	London
600	Ostracon table of numbers, 6221 (Coptic)	Manchester
750	Akhmîm papyrus (Greek)	Cairo
1000	Parchment palimpsest Or 5707, no. 528 in catalogue (demotic and Greek)[3]	London

[1] Wholly unpublished.
[2] Later to be Berkeley.
[3] Mostly untranslated and only partially transcribed.

GENERAL INDEX

Headings of the subdivisions of the Introduction and Free Translation are not given here. A few references are given to matters of interest in the Bibliography, but names of authors and publications and a fuller list of subjects will be found in the indices to the Bibliography itself.